Working Paper Series E-2010-02

How Important is the Fiscal Policy Cooperation in a Currency Union?

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EK-112xvii

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This Draft: June, 2010

*I would like to thank Jacopo Cimadomo, Masaya Ishikawa, Futoshi Kurokawa, Chikafumi Nakamura, Masao Ogaki, Eiji Ogawa, Rui Ota, Nao Sudo, Takayuki Tsuruga, Hugh Patrick, Muhammad Rahman, Marcos P. Ribeiro, Etsuro Shioji, Casper George de Vries, Tsutomu Watanabe, David Weinstein, Taiyo Yoshimi and seminar and conference participants at Columbia University, Federal Reserve Bank of Atlanta, Hitotsubashi University, Maastricht University, Osaka University, Senshu University, University of Crete and University of Ottawa for their useful comments. I have benefited from answers to my questions related to this paper from Andrea Ferrero and Michael Woodford and from valuable comments from Jaromir Nosal. Two anonymous referees' critical comments improved this paper. All errors are my own. This research was assisted by a grant from the Abe Fellowship Program administered by the Social Science Research Council and the American Council of Learned Societies in cooperation with and with funds provided by the Japan Foundation Center for Global Partnership.

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Abstract

By constructing a dynamic stochastic general equilibrium model, which assumes a currency union consisting of two countries with nontradables, we verify the importance of fiscal policy cooperation. As shown in previous studies, we find that the role of fiscal policy is important in maximizing social welfare. However, we have a contrary result for fiscal policy cooperation. While the previous studies highlight that fiscal policy cooperation has a nontrivial effect in maximizing social welfare, we show that fiscal policy cooperation has no benefits regardless of the share of nontradables. Self-oriented fiscal policy can replicate social welfare under the cooperative setting.

Keywords: currency union, DSGE, policy cooperation, optimal monetary policy, monetary and fiscal policy mix *JEL Classification: E52; E62; F41*

1 Introduction

Although widely discussed in the literature on money and finance and international macroeconomics, currency unions still raise new issues in these fields. The establishment of the European Monetary Union (EMU), which is the largest and most important, provides researchers with important research agendas.[1]

For member countries of a currency union, who can no longer conduct their own monetary policy, the role each country's fiscal policy plays as a stabilization or social welfare-maximizing tool is an important issue and has been discussed in detail by many authors. Assuming a lump-sum tax, Benigno[5] analyzes optimal monetary policy in a two-country model with perfect risk sharing. He implies that a central bank within a currency union can achieve welfare maximization not only union-wide but also in each country without support from a fiscal authority. After introducing some frictions, this implication changes. Assuming a currency union consisting of an infinite number of countries, Gali and Monacelli[19] insist on a monetary and fiscal policy mix to maximize social welfare. Under this framework, the central bank can maximize union-wide welfare, whereas it needs strong support from the fiscal authorities to maximize welfare in each country. Introducing rule-of-thumb consumers, who are constrained to spending out of their current disposal income, Colciago, Ropel, Muscatelli and Tirelli[14] find that fiscal policy plays a role not only as a surrogate for the loss of nominal exchange rate flexibility, but also as a stabilization tool for rule-of-thumb consumers' consumption. Ferrero[16] analyzes optimal monetary and fiscal policy in a two-country currency union with a distorted steady state, and finds that optimal fiscal policy is essential in a currency union to maximize social welfare. While these papers only assume tradables, Duarte and Wolman[15] introduce nontradables in their currency union model and show that inflation differentials can be stabilized by adjusting taxation, although they did not consider optimization problems for policy authorities. To summarize the policy implications of these previous studies, fiscal policy is important for stabilizing an economy or enhancing social welfare in a currency union under assumptions that take into account the real economy.

After accepting the importance of fiscal policy in a currency union, we now focus on how to conduct fiscal policy. In particular, discussion on fiscal policy cooperation is not trivial because the EMU consists of many countries. At present, only Beetsma and Jensen[3] have clear policy implications on this topic; fiscal policy cooperation is essential and important for enhancing social welfare via avoiding Nash equilibria brought about by noncooperative fiscal authorities.[2]

It cannot be said that the importance of fiscal policy cooperation is an established policy implication with no opportunity for further discussion because only Beetsma and Jensen[3] show the importance of fiscal policy cooperation. However, the Maastricht Treaty and the Stability and Growth Pact provide the

[1] According to Rose[30], the EMU is the largest and most important currency union.

[2] Duarte and Wolman[15], analyzing fiscal policy in a currency union with nontradables, left an analysis of fiscal policy cooperation as a future research agenda.

legal foundations for organizing fiscal cooperation in the EMU.[3] According to Beetsma and Jensen[3], there is pressure for more fiscal cooperation in Europe. In addition, the fiscal crisis in Greece in March 2010 urges the member countries of the EMU implementing cooperative financial assistance, and provides policy authorities with motivation to discuss the necessity of fiscal policy cooperation. There is room for fuller discussion of fiscal policy cooperation in a currency union because so far only Beetsma and Jensen[3] have analyzed it clearly, and such a discussion is an urgent task for researchers because of the current situation in Europe.

How important is fiscal policy cooperation in a currency union? To answer this question, we construct a dynamic stochastic general equilibrium (DSGE) model with a currency union consisting of two countries with nontradables. Introducing nontradables is not trivial because Beetsma and Jensen[3] assume only tradables. Using this model, we analyze two policy regimes: (i) optimal monetary policy alone, where the central bank conducts monetary policy to enhance social welfare; and (ii) the optimal monetary and fiscal policy mix, where not only the central bank but also local governments in two countries seek to enhance social welfare, under both cooperative fiscal authorities and self-oriented fiscal authorities. In addition, we solve an optimization problem based on a well-microfounded loss function and conduct welfare analysis, which Duarte and Wolman[15] who develop a currency union model with nontradables to analyze fiscal policy, leave for future research.

The answer to our question is that there are no gains from fiscal policy cooperation. Interestingly, our policy implication is not dependent on the share of nontradables. Whether there are nontradables or not, self-oriented fiscal policy can replicate the allocation brought about by a cooperative setting. This implication is contrary to Beetsma and Jensen[3]. In addition, it is contrary to Liu and Pappa[23], who show the importance of policy cooperation in a two-country model with nontradables under a flexible exchange rate. Our policy implication is novel compared with previous studies. In addition, our policy implication does not necessarily depend on parameter values, which removes fiscal authorities' incentive to change the terms of trade (TOT), but may depend on the choice of stabilization tool for fiscal authorities. In a later section in this paper, we discuss why our policy implications do not depend on parameter values, and compare the results of our paper with those derived by previous papers.

Before we analyze fiscal policy cooperation, we show that as the share of nontradables increases, the role of optimal fiscal policy in enhancing social welfare becomes more important. Nontradables lead to consumption disparity between two countries, which stems from the consumer price index (CPI) disparity

[3]Beetsma and Jensen[3] do not mention the plausibility of the Stability and Growth Pact itself. Canzoneri, Cumby and Diba[12] discuss fiscal discipline and exchange rate systems and conclude that to maintain a common currency union, the discipline of a Ricardian regime is essential. They also point out that the fiscal constraint written into the Stability and Growth Pact is sufficient for a Ricardian regime, although they do not mention fiscal policy cooperation.

between the two countries. That nontradables causes the CPI disparity is consistent with Lipinska[22], who analyzes optimal monetary policy in countries wanting to join the EMU, and shows that nontradables' productivity shocks lead to a stronger real exchange rate depreciation. This is the reason why fiscal policy is important provided it is used to stabilize each economy by mitigating the CPI disparity.

The paper is organized as follows. Section 2 constructs the model. Section 3 presents a log-linearized version of the model. Section 4 analyzes a role of optimal fiscal policy under a cooperative setting. Section 5 considers the self-oriented policy settings needed to secure the allocation brought about by a cooperative solution. Section 6 concludes the paper. The technical details are derived in appendix.

2 The Model

We construct a closed-system currency union model belonging to the class of DSGE models with nominal rigidities and imperfect competition, following Obstfeld and Rogoff[26] and Gali and Monacelli[18]. The union-wide economy consists of two equally sized countries, countries H and F. Country H produces an array of differentiated goods indexed by the interval $h \in [0,1]$, while country F produces an array of differentiated goods indexed by $f \in [1,2]$.

2.1 Households

The preferences of the representative household in country H are given by:

$$\mathcal{U} \equiv \mathrm{E}_0 \sum_{t=0}^{\infty} \delta^t U_t, \tag{1}$$

where $U_t \equiv \ln C_t - \frac{1}{1+\varphi} N_t^{1+\varphi}$ denotes the period utility in country H; E_t denotes the expectation, conditional on the information set at period t; $\delta \in (0,1)$ denotes the subjective discount factor; C_t denotes consumption in country H; $N_t \equiv N_{H,t} + N_{\mathcal{N},t}$ denotes hours of work in country H; $N_{H,t}$ and $N_{\mathcal{N},t}$ denote hours of work to produce tradables produced in country H and nontradables produced in country H, respectively; and φ denotes the inverse of the labor supply elasticity. The preferences of the representative household in country F is defined analogously. Quantities and prices particular to country F are denoted by asterisks while quantities and prices without asterisks are those in country H.

More precisely, private consumption is a composite index defined by:

$$C_t \equiv \left[\gamma^{\frac{1}{\eta}} C_{\mathcal{T},t}^{\frac{\eta-1}{\eta}} + (1-\gamma)^{\frac{1}{\eta}} C_{\mathcal{N},t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, \tag{2}$$

where $C_{\mathcal{T},t} \equiv 2 C_{H,t}^{\frac{1}{2}} C_{F,t}^{\frac{1}{2}}$ denotes the consumption index for tradables; $C_{H,t}$, $C_{F,t}$ and $C_{\mathcal{N},t}$ denote Dixit–Stiglitz-type indices of consumption across the tradables produced in country H and produced in country F, and nontradables produced in country H, respectively; $\gamma \in [0,1]$ denotes the share of tradables in the CPI; and $\eta > 0$ denotes the elasticity of substitution between tradables and nontradables. Note that C_t^* is defined analogously to Eq.(2) whereas $C_{\mathcal{N},t}^*$, denoting the nontradables produced in country F replaces $C_{\mathcal{N},t}$.[4]

Total consumption expenditures by households in country H are given by $P_{H,t}C_{H,t}+P_{F,t}C_{F,t}+P_{\mathcal{N},t}C_{\mathcal{N},t} = P_tC_t$, with $P_{H,t}$ and $P_{F,t}$ being Dixit–Stiglitz-type indices of the price of tradables produced in countries H and F, respectively, and $P_{\mathcal{N},t}$ being Dixit–Stiglitz-type indices of the price of nontradables produced in country H. A sequence of budget constraints in country H is given by:

$$D_t^n + W_tN_t + S_t \geq P_tC_t + \mathrm{E}_tQ_{t,t+1}D_{t+1}^n, \tag{3}$$

where $Q_{t,t+1}$ denotes the stochastic discount factor, D_t^n denotes the nominal payoff of the bond portfolio purchased by households, W_t denotes the nominal wage, and S_t denotes profits (net taxation) from ownership of the firms. The budget constraint in country F is defined analogously. Furthermore:

$$P_t \equiv \left[\gamma P_{\mathcal{T},t}^{1-\eta} + (1-\gamma)\, P_{\mathcal{N},t}^{1-\eta}\right]^{\frac{1}{1-\eta}}, \tag{4}$$

denotes the CPI, $P_{\mathcal{T},t} \equiv P_{H,t}^{\frac{1}{2}}P_{F,t}^{\frac{1}{2}}$ denotes the price index of tradables. P_t^* is defined analogously to Eq.(4), whereas $P_{\mathcal{N},t}^*$, denoting the price of nontradables in country F, replaces $P_{\mathcal{N},t}$. We assume that the law of one price always holds, thus $P_{H,t} = P_{H,t}^*$ and $P_{F,t} = P_{F,t}^*$, implying that the prices of tradables are equal in both countries. However, $P_{\mathcal{N},t}$ and $P_{\mathcal{N},t}^*$ are not necessarily equal in both countries because these represent the prices of different goods. These facts imply that purchasing power parity (PPP) does not necessarily hold. When all goods are tradable, Eq.(4) implies $P_t = P_t^*$; namely, PPP always holds.

The optimal allocation of any given expenditure within each category of goods implies the demand functions, as follows:

$$\begin{aligned} C_{H,t} &= \frac{1}{2}\left(\frac{P_{H,t}}{P_{\mathcal{T},t}}\right)^{-1} C_{\mathcal{T},t} \ ; \ C_{F,t} = \frac{1}{2}\left(\frac{P_{F,t}}{P_{\mathcal{T},t}}\right)^{-1} C_{\mathcal{T},t}, \\ C_{\mathcal{T},t} &= \gamma\left(\frac{P_{\mathcal{T},t}}{P_t}\right)^{-\eta} C_t \ ; \ C_{\mathcal{N},t} = (1-\gamma)\left(\frac{P_{\mathcal{N},t}}{P_t}\right)^{-\eta} C_t. \end{aligned} \tag{5}$$

The representative household maximizes Eq.(1) subject to Eq.(3). The optimality conditions are given by:

$$\delta \mathrm{E}_t\left(\frac{C_{t+1}^{-1}P_t}{C_t^{-1}P_{t+1}}\right) = \frac{1}{R_t} \ ; \ C_tN_t^{\varphi} = \frac{W_t}{P_t}, \tag{6}$$

[4] Following Stockman and Tesar[32], we assume that η is not necessarily unity, whereas Obstfeld and Rogoff[26] implicitly assume that η is unity. Obstfeld and Rogoff[26] assume $C_t \equiv \frac{C_{\mathcal{T},t}^{\gamma}C_{\mathcal{N},t}^{1-\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}}$. This implies $\eta = 1$ in our paper.

where $R_t \equiv 1 + r_t$ satisfying $R_t^{-1} = \mathrm{E}_t Q_{t,t+1}$ denotes the gross nominal return on a riskless one-period discount bond paying off one unit of the common currency (in short, the gross nominal interest rate), and r_t denotes the net nominal interest rate. The first equality in Eq.(6) is an intertemporal optimality condition, namely the Euler equation, and the second equality in Eq.(6) is an intratemporal optimality condition. Optimality conditions in country F are given analogously.

Combining and iterating the first equality in Eq.(6) with an initial condition, we have the following optimal risk-sharing condition:

$$C_t = \vartheta C_t^* \mathsf{Q}_t, \tag{7}$$

with $\mathsf{Q}_t \equiv \frac{P_t^*}{P_t}$ denoting the CPI differential between the two countries and ϑ denoting a constant depending on the initial value. When $C_{-1} = C_{-1}^* = P_{-1} = P_{-1}^* = 1$, we have $\vartheta = 1$.[5] In addition, we have $C_t = C_t^*$ which implies that the marginal utility of consumption is equal between the two countries; namely, consumption is equal between the two countries because of the logarithmic utility function when PPP is applied. Our paper allows the existence of nontradables and if there are nontradables, $C_t = C_t^*$ no longer applies because $\mathsf{Q}_t \neq 1$. In that case, there is consumption disparity between the two countries and the single central bank can no longer stabilize both countries simultaneously. Hence the necessity of using fiscal policy to stabilize the economy of each country if there are nontradables. This is an intuitive explanation as to why fiscal policy is important in a currency union with nontradables.

Eq.(7) is useful for understanding intuitively why Gali and Monacelli[19] insist on the importance of fiscal policy although they only assume tradables in a currency union that consists of an infinite number of infinitesimally small countries. Because the home country is infinitesimally small, the share of the home country price index in the union-wide price index is negligible. The marginal utility of consumption between the home country and the union as a whole is not equal and there is opportunity to conduct fiscal policy independently in each country.[6]

[5] See Chari, Kehoe and McGrattan[13] for details.

[6] The reason why fiscal policy is important in a currency union is consistent with this paper and Gali and Monacelli[19]; however, the stabilization tools and settings in these two papers are clearly different. We assume that the fiscal authorities adjust the quantity of government bonds, while Gali and Monacelli[19] assume that the fiscal authorities adjust government expenditure. In their setting, fiscal authorities' funds are lump-sum taxes, while in our setting, fiscal authorities' funds are obtained by issuance of government bonds. Such new bond issues change the distribution of private savings and that may change output and inflation. In Gali and Monacelli[19]'s lump-sum taxes setting, how the government secures funds for spending is not clear because such a setting does not show an explicit government budget constraint. In contrast, we show clearly how to manage government debt to maximize social welfare by introducing explicitly a government budget constraint.

2.2 Firms

Each producer uses a linear technology to produce a differentiated good as follows:

$$Y_{H,t}(h) = A_{H,t} N_{H,t}(h), \quad ; \quad Y_{\mathcal{N},t}(h) = A_{\mathcal{N},t} N_{\mathcal{N},t}(h), \tag{8}$$

where $Y_{H,t}(h)$ and $Y_{\mathcal{N},t}(h)$ denote the output of tradables h produced in country H, and output of nontradables h produced in country H, respectively, and $A_{H,t}$ and $A_{\mathcal{N},t}$ denote stochastic productivity shifters associated with tradables produced in country H and nontradables produced in country H, respectively. Each producer in country F uses a technology similar to that in country H.

Each firm produces a single differentiated good and prices its good to reflect the elasticity of substitution across goods produced given the CPI. This is because each firm plays an active part in the monopolistically competitive market. We assume that Calvo–Yun-style price-setting behavior applies, and, therefore, that each firm resets its price with a probability of $1 - \alpha$ in each period, independently of the time elapsed since the last adjustment.

When setting a new price in period t, firms seek to maximize the expected discounted value of profits. The first-order necessary conditions (FONCs) are as follows:

$$\begin{aligned}
\mathrm{E}_t\left[\sum_{k=0}^{\infty}(\alpha\delta)^k\left(P_{t+k}C_{t+k}\right)^{-1}\tilde{Y}_{H,t+k}\left(\tilde{P}_{H,t} - \zeta MC^n_{H,t+k}\right)\right] &= 0, \\
\mathrm{E}_t\left[\sum_{k=0}^{\infty}(\alpha\delta)^k\left(P_{t+k}C_{t+k}\right)^{-1}\tilde{Y}_{N,t+k}\left(\tilde{P}_{\mathcal{N},t} - \zeta MC^n_{N,t+k}\right)\right] &= 0,
\end{aligned} \tag{9}$$

where $MC^n_{H,t} \equiv \frac{W_t}{(1-\tau)A_{H,t}}$ and $MC^n_{\mathcal{N},t} \equiv \frac{W_t}{(1-\tau)A_{\mathcal{N},t}}$ denote the nominal marginal costs associated with tradables produced in country H and nontradables produced in country H, respectively; $\tilde{Y}_{H,t}$ and $\tilde{Y}_{\mathcal{N},t}$ denote the total demands following changes in the prices of tradables produced in country H and nontradables produced in country H, respectively; $\tilde{P}_{H,t}$ and $\tilde{P}_{\mathcal{N},t}$ denote the adjusted prices of tradables produced in country H and nontradables produced in country H, respectively; and $\zeta \equiv \frac{\theta}{\theta-1}$ is a constant markup, $\theta > 1$ denotes the elasticity of substitution across goods produced within a country and τ denotes the tax rate.[7] Note that $\left(P_{t+k}C_{t+k}\right)^{-1}$ is the marginal utility of nominal income.

2.3 Local Government

Whereas monetary frictions are omitted and the limitations of a "cashless economy" are considered following Woodford[35] throughout this paper, monetary policy has important implications for fiscal decisions, as the level of the interest rate determines the debt burden and the inflation rate affects the real value of debt. Fiscal policy consists of choosing one-period nominal risk-free debt to

[7] Ferrero[16] regards it as a value-added tax rate.

finance an exogenous process of public spending.[8] The flow government budget constraint in country H is given by:

$$B_t^n = R_{t-1}B_{t-1}^n - \left[P_{P,t}\tau\left(Y_{H,t} + Y_{\mathcal{N},t}\right) - P_{G,t}\left(G_{H,t} + G_{\mathcal{N},t}\right)\right], \tag{10}$$

where $B_t^n \equiv P_t B_t$ denotes the nominal risk-free rate on bonds issued by the local government in country H; B_t denotes the real risk-free rate on bonds issued by the local government in country H; $P_{P,t} \equiv \frac{P_{H,t}Y_{H,t}+P_{\mathcal{N},t}Y_{\mathcal{N},t}}{Y_{H,t}+Y_{\mathcal{N},t}}$ denotes the weighted average price of goods produced in country H, namely, the producer price index (PPI) in country H; and $P_{G,t} \equiv \frac{P_{H,t}G_{H,t}+P_{\mathcal{N},t}G_{\mathcal{N},t}}{G_{H,t}+G_{\mathcal{N},t}}$ denotes the average price of goods purchased by the government in country H. The local government in country F has a budget constraint similar to that shown in Eq.(10). For simplicity, we assume that government purchases are fully allocated to a domestically produced good and that the total amount of these is exogenous. For any given level of public consumption, the government allocates expenditures across goods in order to minimize total cost. This equation becomes the New Keynesian IS (NKIS) curve after being combined with Eq.(6) and log-linearized with the appropriate transversality condition.[9]

2.4 Market Clearing

The market for tradables and for nontradables in country H clears when domestic demand equals domestic supply, as follows:

$$Y_{H,t}(h) = C_{H,t}(h) + C_{H,t}^*(h) + G_{H,t}(h) \;\; ; \;\; Y_{\mathcal{N},t}(h) = C_{\mathcal{N},t}(h) + G_{\mathcal{N},t}(h), \tag{11}$$

where $C_{H,t}^*(h)$ denotes country F's demand for generic tradables produced in country H.

Finally, we define countrywide output and government expenditure as follows:

$$Y_t \equiv \frac{P_{H,t}}{P_{P,t}}Y_{H,t} + \frac{P_{\mathcal{N},t}}{P_{P,t}}Y_{\mathcal{N},t} \quad ; \quad G_t \equiv \frac{P_{H,t}}{P_{G,t}}G_{H,t} + \frac{P_{\mathcal{N},t}}{P_{G,t}}G_{\mathcal{N},t}, \tag{12}$$

where Y_t and G_t denote output and government expenditure in country H, respectively.

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8 We assume the tax rate is common in each country and constant over time for simplicity, whereas Ferrero[16], Schmitt-Grohe and Uribe[31] and Canzoneri, Cumby and Diba[12] assume it can vary in each country and over time. In contrast, Gali and Monacelli[19] assume a constant (negative) tax rate similar to our paper. A constant tax rate over time reflects actual regimes, not only in the Euro area, but also in other countries, because adjustments of the tax rate are infrequent.

9 The transversality condition is given by $\lim_{k\to\infty} \mathrm{E}_t \left[Q_{t,k}\frac{1}{2}\left(B_k^n + B_k^{*n}\right)\right] = 0$ and $\lim_{k\to\infty} \mathrm{E}_t \left[\delta^{k-t}U_C(C)RB\right] = 0$ with $B = B^* > 0$, which are consistent with Ferrero[16] where variables without a time subscript denote steady-state values.

3 Log-linearized Model

This section describes the stochastic equilibrium that arises from perturbations around the deterministic equilibrium. Lowercase letters denote percentage deviations of steady-state values for the respective uppercase letters when there is no note to the contrary (i.e., $v_t \equiv \frac{dV_t}{V}$, where V_t denotes an arbitrary variable and V denotes the steady-state value of V_t). Lowercase letters accompanied by R as a superscript indicate the logarithmic difference between the two countries for the respective uppercase letters (i.e., $v_t^R \equiv v_t - v_t^*$), while lowercase letters accompanied by W as a superscript indicate the logarithmic weighted sum of the two countries for the respective uppercase letters (i.e., $v_t^W \equiv \frac{1}{2}(v_t + v_t^*)$).

Combining Eqs.(4), (5) and (10)–(12) and the log-linear approximation, we have NKISs as follows:

$$\begin{aligned}
\tilde{y}_t^W &= \frac{\beta_W}{1-\sigma_G}\mathrm{E}_t\tilde{y}_{t+1}^W - \beta_W\hat{r}_t + \beta_W\mathrm{E}_t\pi_{t+1}^W + \frac{\beta_W}{\delta}\hat{r}_{t-1} - \beta_W b_t^W + \frac{\beta_W}{\delta}b_{t-1}^W, \\
&\quad - \frac{\beta_W}{\delta}\pi_t^W - \frac{\gamma\bar{\beta}\beta_{\mathcal{T}}}{2}a_{H,t} - \frac{(1-\gamma)\bar{\beta}\beta_{\mathcal{N}}}{2}a_{\mathcal{N},t} - \frac{\gamma\bar{\beta}\beta_{\mathcal{T}}}{2}a_{F,t}, \\
&\quad - \frac{(1-\gamma)\bar{\beta}\beta_{\mathcal{N}}}{2}a_{\mathcal{N},t}^* + \sigma_G\varsigma_W g_t^W, \qquad (13) \\
\tilde{y}_t^R &= -\beta_R\delta b_t^R + \beta_R(1-\gamma)\upsilon\mathsf{n}_t - \beta_R(1-\gamma)\mathsf{n}_{t-1} + \beta_R b_{t-1}^R - \bar{\beta}\gamma a_{H,t}, \\
&\quad + \bar{\beta}\gamma a_{F,t} - \bar{\beta}(1-\gamma)a_{\mathcal{N},t} + \bar{\beta}(1-\gamma)a_{\mathcal{N},t}^* + \varsigma_R\sigma_G g_t^R, \qquad (14)
\end{aligned}$$

where $\tilde{y}_t \equiv y_t - \bar{y}_t$ denotes the logarithmic output gap measured from its natural level in country H and $\bar{y}_t$ denotes the logarithmic natural output level in country H, which becomes $\tilde{y}_t = 0$ under the long run equilibrium; $\hat{r}_t \equiv \frac{dR_t}{R}$ denotes the deviation of the nominal interest rate from its steady-state value; π_t denotes the CPI inflation rate in country H; n_t denotes the logarithmic nontradables price disparity between countries H and F (NPD) with $\mathsf{N}_t \equiv \frac{P_{\mathcal{N},t}^*}{P_{\mathcal{N},t}}$, $\beta_W \equiv \frac{(1-\sigma_G)\sigma_B}{\sigma_B+(1-\sigma_G)\tau}$, $\beta_R \equiv \frac{\sigma_B(1-\sigma_G)}{(1-\sigma_G)\delta\tau-(1-\delta)\sigma_B}$, $\upsilon \equiv 1-(1-\delta)\varpi$, $\varpi \equiv 1+(\eta-1)\gamma$, $\nu_W \equiv \frac{[\sigma_B(1-\rho_G)+1-\sigma_G]}{\sigma_B+(1-\sigma_G)\tau}$, $\nu_R \equiv \frac{[(1-\sigma_G)\delta-(1-\delta)\sigma_B]}{(1-\sigma_G)\delta\tau-(1-\delta)\sigma_B}$, $\beta_{\mathcal{T}} \equiv 1-\frac{\beta_W\rho_{\mathcal{T}}}{1-\sigma_G}$, $\beta_{\mathcal{N}} \equiv 1-\frac{\beta_W\rho_{\mathcal{N}}}{1-\sigma_G}$, $\bar{\beta} \equiv \frac{(1-\sigma_G)(1+\varphi)}{\lambda}$, $\varsigma_W \equiv \nu_W + \frac{\beta_W\rho_G}{(1-\sigma_G)\lambda} - \frac{1}{\lambda}$ and $\varsigma_R \equiv \nu_R - \frac{1}{\lambda}$, $\sigma_B \equiv \frac{B}{Y}$ and $\sigma_G \equiv \frac{G}{Y}$ being the steady-state ratio of government bonds to output and the steady-state ratio of government expenditure to output, respectively; and $\rho_G < 1$, $\rho_{\mathcal{T}} < 1$ and $\rho_{\mathcal{N}} < 1$ being the coefficient associated with exogenous processes on government expenditure, on the productivity shifter of tradables and on the productivity of nontradables, respectively.[10] When all goods are tradables, $\gamma = 1$ is applied and Eq.(14) equals: $\tilde{y}_t^R = -\beta_R\delta b_t^R + \beta_R b_{t-1}^R - \gamma a_{H,t} + \gamma a_{F,t} + \varsigma_R\sigma_G g_t^R$, in which not only the productivity of nontradables $a_{\mathcal{N},t}$ and $a_{\mathcal{N},t}^*$, but also the NPD n_t disappears. The appearance of the NPD in the equality implies that nontradables magnifies the output gap disparity between the two countries although risk sharing is perfect internationally. This is the reason why a higher

[10] We assume that the government expenditure and productivity shifters follow AR(1) processes (see Subsection 4.2.)

share of nontradables increases the importance of the role of fiscal policy in maximizing social welfare. In addition, we can understand why Ferrero[16] highlights the importance of optimal fiscal policy in a currency union, although he does not assume nontradables. When the steady-state value of the quantity of government bonds and expenditure are zero, $\sigma_B = \sigma_G = 0$, which implies an efficient steady state and this is assumed by Benigno[5]. In this case, Eq.(14) becomes: $\tilde{y}_t^R = -\gamma a_{H,t} + \gamma a_{F,t}$, which implies that only productivity affects the output gap disparity.[11] Following Ferrero[16], we assume a distorted steady state where $\sigma_B = \sigma_G = 0$ is not applied. In this case, relative government expenditure affects the output gap disparity and the role of fiscal policy is larger than the case of $\sigma_B = \sigma_G = 0$, although there are no nontradables.

Combining Eqs.(5)–(9), (11) and (12) and taking a log-linear approximation, the NKPCs in terms of the output gap are given by:

$$\pi_{P,t} = \delta \mathrm{E}_t \pi_{P,t+1} + \frac{\kappa\lambda}{1-\sigma_G}\tilde{y}_t - \frac{\psi\kappa}{2}\mathsf{n}_t, \tag{15}$$

where $\pi_{P,t}$ denotes the PPI inflation rate in country H with $\kappa \equiv \frac{(1-\alpha)(1-\alpha\delta)}{\alpha}$, $\lambda \equiv 1+(1-\sigma_G)\varphi$ and $\psi \equiv (1-\gamma)\gamma(\eta-1)$ along with its counterpart in country F. When $\gamma = 1$, Eq.(15) is written as $\pi_{P,t} = \delta \mathrm{E}_t \pi_{P,t+1} + \frac{\kappa\lambda}{1-\sigma_G}\tilde{y}_t$, which corresponds with that derived by Gali and Monacelli[18], who insist that inflation–output tradeoffs can be dissolved simultaneously in a small open economy under strong parameter restrictions by inflation targeting. Indeed, when inflation targeting, such as $\pi_{P,t} = \pi_{P,t}^* = 0$ for all t, is introduced in our currency union with special restrictions, i.e., $\gamma = 1$ and $\sigma_B = \sigma_G = 0$, these equalities imply that $\tilde{y}_t = \tilde{y}_t^* = 0$ for all t and that the output gap is eliminated.

Combining Eqs.(5)–(9), (11) and (12) and taking a log-linear approximation, we have the New Keynesian real exchange rate determiner (NKRD), which is our version of the real exchange rate determiner, as follows:

$$\begin{aligned} \pi_{\mathcal{N},t}^R \quad = \quad & \delta \mathrm{E}_t \pi_{N,t+1}^R + \kappa\varphi\tilde{y}_t^R + \kappa\mathsf{n}_t - \kappa\varphi\gamma\left(1-\bar{\beta}\right)a_{H,t} + \kappa\varphi\gamma\left(1-\bar{\beta}\right)a_{F,t}, \\ & - \kappa\left[1+\varphi(1-\gamma)\left(1-\bar{\beta}\right)\right]a_{\mathcal{N},t} + \kappa\left[1+\varphi(1-\gamma)\left(1-\bar{\beta}\right)\right]a_{\mathcal{N},t}^*, \\ & - \frac{\kappa\sigma_G}{1-\sigma_G}\left(1-\frac{\varphi}{\lambda}\right)g_t^R, \end{aligned} \tag{16}$$

where $\pi_{\mathcal{N},t}^R \equiv -(\mathsf{n}_t - \mathsf{n}_{t-1})$ denotes relative nontradables inflation. Eq.(16) equals $\mathsf{q}_t = 0$ which implies that the CPI disparity disappears between the two countries when the currency union has no nontradables; i.e., as $\gamma = 1$.[12] In that case, the problem with the CPI disparity is resolved, because each country has the same CPI. This implies that PPP holds. Furthermore, because log-linearized Eq.(7) is given by $\mathsf{q}_t = c_t^R$, $c_t^R = 0$, which means there is no consumption disparity between both countries when $\mathsf{q}_t = 0$.

Benigno[5] and Gali and Monacelli[19] assume that all goods are tradable and that the law of one price holds. However, PPP does not necessarily hold in Gali

[11] In addition, $y_t^R = 0$, which implies that there is no output disparity in that case.
[12] We obtain this result by substituting log-linearized Eq.(4) into Eq.(16).

and Monacelli[19]. They assume a currency union that consists of an infinite number of countries, whereas Benigno[5] assumes a currency union consisting of two countries. The settings in Gali and Monacelli[18] make a distinction in the CPI between that of an infinitesimally small country's economy and that of the union-wide economy because the CPI in the former country does not affect the union-wide CPI. This stems from the small open economy assumption. Thus, $\mathsf{q}_t = 0$ is not applied in Gali and Monacelli[19], although all goods are tradable. In a later section, we suggest that optimal fiscal policy is very important for stabilizing both inflation and the output gap simultaneously because of the influence of the nontradables. The assumption of a small open economy does not permit us to apply $\mathsf{q}_t = 0$. Thus, the policy implications of Gali and Monacelli[19] and this paper are very similar, whereas the policy implications of Gali and Monacelli[19] and Benigno[5] are contrary.

Eq.(16) depicts the real exchange rate anomaly, which is reported by Benigno and Thoenissen[8] and Canzoneri, Cumby and Diba[11], who find that the actual direction of changes in the real exchange rate cannot be explained by the Balassa–Samuelson theorem. Eq.(16) well reflects their findings in a well-founded micro setting. However, it cannot be easily understood because Eq.(16) has correctly assumed nominal rigidities. To understand the nature of Eq.(16), we inspect Eq.(16) without nominal rigidities. Under such a condition, Eq.(16) can be rewritten as:

$$\begin{aligned}\mathsf{q}_t \quad = \quad & (1-\gamma)\left\{\varphi\gamma\left(1-\bar{\beta}\right)a_{H,t} + \left[1+\varphi\left(1-\gamma\right)\left(1-\bar{\beta}\right)\right]a_{\mathcal{N},t} - \varphi\gamma\left(1-\bar{\beta}\right)a_{F,t}\right. \\ & \left. - \left[1+\varphi\left(1-\gamma\right)\left(1-\bar{\beta}\right)\right]a^*_{\mathcal{N},t} - \frac{\varphi\sigma_G}{1+\varphi}g^R_t\right\}, \qquad (17)\end{aligned}$$

because $\alpha = 0$ and $\tilde{y}_t = \tilde{y}^*_t = 0$ hold. Eq.(17) implies that an increase in the productivity of tradables produced in country H causes an increase (depreciation) in the real exchange rate. An increase in the productivity of tradables produced in country H causes a decrease in the CPI in coutry H via a decrease in the real marginal cost in the tradables and the nontradables sectors. Finally, the real exchange rate increases (depreciates).

In addition, our NKRD is consistent with Altissimo, Benigno and Palenzuela [1]'s equality, which determines the real exchange rate and corresponds to our NKRD. Their equality shows that the productivity shifter of tradables does not affect the real exchange rate and only the productivity shifter of nontradables affects the real exchange rate, under their special setting where the relative risk aversion, the elasticity of substitution between tradables and nontradables, and the elasticity of substitution between tradables produced in countries H and F are all unity. Their special setting is consistent with our model, because we assume Eq.(1) and an Armington form of the definition of the consumption index for tradables and the elasticity of substitution between tradables and nontradables does not appear in Eq.(16). Altissimo, Benigno and Palenzuela [1] assume zero steady-state government expenditure, which corresponds to $G = 0$.

Substituting $G = 0$ into Eq.(17) yields:

$$\mathsf{q}_t = (1-\gamma)\left[a_{\mathcal{N},t} - a^*_{\mathcal{N},t} - \frac{\varphi}{1+\varphi}\left(\frac{dG_t}{Y}\right)^R\right], \tag{18}$$

which is consistent with the equilibrium condition on the CPI differential under the special case corresponding to our setting derived by Altissimo, Benigno and Palenzuela [1], who said that the Balassa–Samuelson effect disappears in that special setting because the productivity shifter of tradables disappears as in Eq.(18).[13]

4 Optimal Cooperative Solution

In this section, we analyze the macroeconomic implications of an alternative policy regime for the Euro area: an optimal monetary policy alone regime and an optimal monetary and fiscal policy regime under a cooperative setting. Furthermore, we assume that each policy authority is responsible for minimizing social losses. Under an optimal monetary policy alone regime, the central bank is the only policy authority, whereas the central bank and local governments in the two countries are both authorities under an optimal monetary and fiscal policy regime. Policy authorities seek to minimize the social loss function subject to our structural model.[14] Hereafter, let us assume $\eta = 1$, implying that the elasticity of substitution between tradables and nontradables is unity, which is assumed implicitly by Obstfeld and Rogoff[26], for simplicity.

The period loss function is derived by the welfare criterion. The welfare criterion is derived by subtracting a second-order approximated FONC for firms Eq.(9) $\mathcal{M}^W$ from a second-order approximated utility function Eq.(1), which is given by:

$$\begin{aligned}\mathcal{W}^W - \Phi\mathcal{M}^W \quad = \quad & \mathrm{E}_0\sum_{t=0}^{\infty}\delta^t\left\{\frac{1}{(1-\sigma_G)\,4}\tilde{\omega}_1\left[\tilde{y}_t^2 + (\tilde{y}_t^*)^2\right] - \tilde{\omega}_2\left(\tilde{y}_t a_t + \tilde{y}_t^* a_t^*\right)\right.\\ & \left. -\tilde{\omega}_3\left(\tilde{y}_t g_t + \tilde{y}_t^* g_t^*\right) + \frac{1}{4}\tilde{\omega}_4\left[\pi_{P,t}^2 + \left(\pi_{P,t}^*\right)^2\right] + \frac{\Phi(1-\kappa\lambda)}{1-\sigma_G}\tilde{y}_t^W\right\},\\ & + \text{t.i.p.} + o\left(\|\xi\|^3\right),\end{aligned}$$

[13] Altissimo, Benigno and Palenzuela [1] show that an increase in the productivity of nontradables in the home (foreign) country increases (decreases) the real exchange rate, while an increase in government expenditure in the home (foreign) country decreases (increases) the real exchange rate when relative risk aversion, the elasticity of substitution between tradables and nontradables, and the elasticity of substitution between tradables produced in countries H and F are all unity. Because we consider a currency union, the logarithmic real exchange rate corresponds to the CPI disparity q_t. Thus, Eq.(18) is consistent with the equation derived by Altissimo, Benigno and Palenzuela [1].

[14] Our structural model consists of Eqs.(13)–(16) and a counterpart of Eq.(15) in country F.

with $\mathcal{W}^W \equiv \mathrm{E}_0 \sum\limits_{t=0}^{\infty} \delta^t \left(U_t^W - U^W\right)$ denoteing the sum of the discounted value of the deviation of utility from its steady-state value, $\Phi \equiv 1 - \frac{1-\tau}{\zeta}$ denoting the steady-state wedge between the marginal rate of substitution between consumption and leisure and the marginal product of labor, t.i.p. denoteing the terms of the independent policy, $o\left(\|\xi\|^3\right)$ denoting the terms that are higher than third order and $\tilde{\omega}_1$, $\tilde{\omega}_2$, $\tilde{\omega}_3$ and $\tilde{\omega}_4$ being complicated coefficients that consists of structural parameters in our model. Plugging the fact that $\mathrm{E}_0 \sum_{t=0}^{\infty} \delta^t \frac{\Phi(1-\kappa\lambda)}{1-\sigma_G} \tilde{y}_t^W + \Phi \mathcal{M}^W = \Gamma_0$ into this equality, we have:

$$\begin{aligned} \mathcal{W}^W &= -\frac{1}{2}\mathrm{E}_0 \sum_{t=0}^{\infty} \delta^t \left[\frac{\Lambda_y}{2}\hat{y}_t^2 + \frac{\Lambda_y}{2}\left(\hat{y}_t^*\right)^2 + \frac{\Lambda_\pi}{2}\pi_{P,t}^2 + \frac{\Lambda_\pi}{2}\left(\pi_{P,t}^*\right)^2\right] + \Gamma_0 + \text{t.i.p.}, \\ &\quad + o\left(\|\xi\|^3\right), \end{aligned}$$

which is the welfare criterion without linear terms where; $\hat{y}_t \equiv y_t - y_t^e$ denotes the welfare-relevant output gap; $y_t^e \equiv \Omega_1 \gamma a_{H,t} + \Omega_1 (1-\gamma) a_{\mathcal{N},t} + \Omega_2 g_t$ denotes the logarithmic efficient level of output in country H; and $\Gamma_0 \equiv \frac{\Phi}{\kappa\lambda}\pi_{P,0}^W$ denotes a transitory component which is predetermined;, with $\Lambda_y \equiv \frac{\chi+\kappa\Phi\varsigma}{(1-\sigma_G)^2}$, $\Lambda_\pi \equiv \frac{(1+\Phi)\theta}{(1-\sigma_G)\kappa} + \frac{\Phi\Theta}{2}$, $\chi \equiv (1+\varphi)(1+\Phi)(1-\sigma_G)$, $\varsigma \equiv (1+\varphi)^2 (1-\sigma_G)^2 - \sigma_G^2$ and $\Theta \equiv \alpha(4\theta - 1) - 3$; and Ω_1 and Ω_2 are complicated coefficients that consist of structural parameters in our model.

Because the transitory component is predetermined over the set of admissible policies, the higher values of this welfare criterion correspond to lower values of the loss function, which is given by:

$$\mathcal{L}^W \equiv \mathrm{E}_0 \sum_{t=0}^{\infty} \delta^t L_t^W, \tag{19}$$

with:

$$L_t^W \equiv \frac{1}{2}\left[\frac{\Lambda_y}{2}\hat{y}_t^2 + \frac{\Lambda_y}{2}\left(\hat{y}_t^*\right)^2 + \frac{\Lambda_\pi}{2}\pi_{P,t}^2 + \frac{\Lambda_\pi}{2}\left(\pi_{P,t}^*\right)^2\right], \tag{20}$$

where L_t^W denotes the union-wide period loss function.[15]

We now consider the difference in welfare loss between our paper and the DSGE literature for an open economy. Our period welfare loss Eq.(20) has two features that do not appear in other period welfare functions. First, Eq.(20) implies that policy authorities should not minimize the output gap $\tilde{y}_t$, but should minimize the welfare-relevant output gap $\hat{y}_t$ because of the distorted steady state

[15] Sutherland[33] and Benigno and Woodford[9] derive the second-order approximated utility function without the presence of a linear term under the distorted steady state. Thus, we follow Benigno and Woodford[10] to derive Eq.(20) because of the distorted steady state in our model. See Appendix D for details on the derivation. Note that Woodford[35] discusses how the presence of linear terms generally leads to an incorrect evaluation of welfare. A simple example of this result is proposed by Kim and Kim[21].

and the nonzero steady state value of government expenditure. When the steady state is efficient and government expenditure is zero in the steady state, namely, both $\Phi = 0$ and $\sigma_G = 0$ are applied, Eq.(20) can be rewritten as follows:

$$L_t^W = \frac{(1+\varphi)}{4}\tilde{y}_t^2 + \frac{(1+\varphi)}{4}\left(\tilde{y}_t^*\right)^2 + \frac{\theta}{4\kappa}\pi_{P,t}^2 + \frac{\theta}{4\kappa}\left(\pi_{P,t}^*\right)^2,$$

which implies that the policy authorities have to minimize the output gap. This expression is similar to Gali and Monacelli[18] and Gali[17]. Note that $\Omega_1 = 1$, $\Omega_2 = 0$ and $y_t^e = \bar{y}_t = \gamma a_{H,t} + (1-\gamma)\, a_{\mathcal{N},t}$ when $\Phi = 0$ and $\sigma_G = 0$.

Second, by rewriting Eq.(20), we can see that nontradables affect the form of the period loss. Using log-linearized Eqs.(5) and (11), Eq.(20) can be rewritten as follows:

$$\begin{aligned} L_t^W &= \Lambda_y\left(\hat{y}_t^W\right)^2 + \frac{\Lambda_y\left[\gamma\left(1-\sigma_G\right)\right]^2}{4}\left(\mathsf{t}_t - \bar{\mathsf{t}}_t\right)^2 + \frac{\Lambda_y\left[\left(1-\gamma\right)\varpi\left(1-\sigma_G\right)\right]^2}{4}\left(\mathsf{n}_t - \bar{\mathsf{n}}_t\right)^2 \\ &\quad + \frac{\Lambda_y 2\gamma\left(1-\sigma_G\right)^2\left(1-\gamma\right)\varpi}{4}\mathsf{t}_t\mathsf{n}_t + \frac{\Lambda_\pi}{2}\pi_{P,t}^2 + \frac{\Lambda_\pi}{2}\left(\pi_{P,t}^*\right)^2, \end{aligned}$$

where t_t denotes the logarithmic TOT with $\mathsf{T}_t \equiv \frac{P_{F,t}}{P_{H,t}}$, $\bar{\mathsf{t}}_t$ and $\bar{\mathsf{n}}_t$ denote the logarithmic target level of the TOT and the NPD, respectively, with $\bar{\mathsf{t}}_t = \bar{\mathsf{n}}_t \equiv \frac{\Omega_1}{1-\sigma_G}a_{H,t} + \frac{\Omega_1(1-\gamma)}{\gamma(1-\sigma_G)}a_{\mathcal{N},t} - \frac{\Omega_1}{1-\sigma_G}a_{F,t} - \frac{\Omega_1(1-\gamma)}{\gamma(1-\sigma_G)}a_{\mathcal{N},t}^* + \frac{\Omega_2-\sigma_G}{\gamma(1-\sigma_G)}g_t^R$ implying that both the target level of the logarithmic TOT and the NPD are equal. As shown in this equality, not only minimization of the distance of the TOT from its target level, but also minimization of the distance of the NPD from its target level are imposed policy objectives. Furthermore, a cross-term of the TOT and the NPD appears in this equality. These facts imply that stabilizing the NPD is essential for minimizing social loss in a currency union with nontradables. However, when all goods are tradable, namely $\gamma = 1$, this equality boils down to:

$$L_t^W = \Lambda_y\left(\hat{y}_t^W\right)^2 + \frac{\Lambda_y\left[\gamma\left(1-\sigma_G\right)\right]^2}{4}\left(\mathsf{t}_t - \bar{\mathsf{t}}_t\right)^2 + \frac{\Lambda_\pi}{2}\pi_{P,t}^2 + \frac{\Lambda_\pi}{2}\left(\pi_{P,t}^*\right)^2,$$

which is a familiar expression in the open economy version DSGE literature, such as Benigno and Benigno[7], assuming all goods are tradable except for the welfare-relevant output gap replacing the output gap. Because our model allows for nontradables, our welfare loss does not necessarily correspond to the welfare loss in other DSGE studies for an open economy.

Note that the linear constraints implied by our model's structural equations in terms of the welfare-relevant output gap differ from the constraints implied by the equations in terms of the output gap. By using the definition of the welfare-relevant output gap, the NKPC in country H Eq.(15) can be written as follows:

$$\pi_{P,t} = \delta \mathrm{E}_t \pi_{P,t+1} + \frac{\kappa\lambda}{1-\sigma_G}\hat{y}_t + \varepsilon_t, \tag{21}$$

where $\varepsilon_t \equiv \kappa(1+\varphi)\Omega_3\gamma a_{H,t} + \kappa(1+\varphi)\Omega_3(1-\gamma)a_{\mathcal{N},t} - \kappa\sigma_G\Omega_4 g_t$ is a composite cost-push term with Ω_3 and Ω_4 denoting complicated coefficients. This NKPC corresponds to the one derived by Benigno and Woodford[10]. A composite cost-push term indicates the degree to which the exogenous disturbances preclude simultaneous dissolution of the tradeoff between inflation and the welfare-relevant output gap. Thus, the inflation–output gap tradeoffs can no longer be dissolved completely, even if all goods are tradable.

4.1 Role of Optimal Fiscal Policy

We next investigate the role of optimal fiscal policy by comparing FONCs, which clarify the relationship between PPI inflation and the output gap under the optimal monetary policy alone, and under the optimal monetary and fiscal policy. Under both regimes, policy authorities minimize the sum of the discounted value of social losses in Eq.(19), subject to the structural model with commitment.[16] Under the optimal monetary policy alone, only the central bank minimizes Eq.(19) by choosing the sequence $\left\{\pi_{P,t}, \pi^*_{P,t}, \tilde{y}_t, \tilde{y}^*_t, \mathsf{n}_t, \hat{r}_t\right\}_{t=0}^{\infty}$, while both the central bank and two local governments cooperatively minimize Eq.(19) by choosing $\left\{\pi_{P,t}, \pi^*_{P,t}, \tilde{y}_t, \tilde{y}^*_t, \mathsf{n}_t, \hat{r}_t, b_t, b^*_t\right\}_{t=0}^{\infty}$ under the optimal monetary and fiscal policy.

4.1.1 Optimal Monetary Policy Alone

Under the optimal monetary policy alone, the FONC for union-wide inflation and the output gap is given by:

$$\pi^W_t = -\frac{\Lambda_y(1-\sigma_G)}{\Lambda_\pi\kappa\lambda}\left(\hat{y}^W_t - \hat{y}^W_{t-1}\right), \tag{22}$$

which is a familiar expression in papers on optimal monetary policy in an open economy. This implies that local government does not need to dissolve union-wide inflation–output tradeoffs. A solitary central bank can stabilize both inflation and the welfare-relevant output gap simultaneously, even though nontradables exist.[17]

Next, we investigate the relative block of the FONC. We are interested in the effects of nontradables. Thus, we analyze the relative block of the FONC in

[16] Because of commitment, lagged Lagrange multipliers appear in the FONCs for policy authorities. This means that policy authorities are able to affect the private sector's inflation expectations. See Appendix E.

[17] Because of the distorted steady state, the NKPC in terms of the welfare-relevant output gap includes a cost-push term as shown in Eq.(21). This term indicates the degree to which the exogenous disturbances preclude the simultaneous dissolution of the tradeoff between inflation and the welfare-relevant output gap. While we have the FONC Eq.(22), the inflation–output gap tradeoffs cannot be completely dissolved but they are close to being dissolved. Under our parameterization, which is shown in Section 4.2.1, the standard deviations of the union-wide inflation rate and the welfare-relevant output gap are 6.3215e-005 and 0.0014, respectively. These standard deviations are equivalent under both optimal monetary policy alone and optimal monetary and fiscal policy.

both cases: namely, the case where all goods are tradable and the case where there are nontradables. When all goods are tradable, namely, $\gamma = 1$, the relative block of the FONC is given by:

$$\pi^R_{P,t} = -\frac{\Lambda_y (1-\sigma_G)}{\Lambda_\pi \kappa \lambda} \left(\hat{y}^R_t - \hat{y}^R_{t-1}\right) - \frac{2(1-\sigma_G)}{\Lambda_\pi \kappa \lambda} \left(\mu_{2,t} - \mu_{2,t-1}\right), \tag{23}$$

where $\mu_{2,t}$ denotes the Lagrange multiplier associated with Eq.(14); that is, the relative block of the NKIS. This case corresponds to Ferrero's (2009) setting. Eq.(23) implies that the inflation–output tradeoffs no longer disappear simultaneously. Because of this, Ferrero[16] insists that fiscal policy is needed to enhance social welfare. Next, we abandon the assumption that all goods are tradable. In this case, the relative block of the FONC is given by:

$$\begin{aligned}
\pi^R_{P,t} &= -\frac{\Lambda_y (1-\sigma_G)}{\Lambda_\pi \kappa \lambda} \left(\hat{y}^R_t - \hat{y}^R_{t-1}\right) - \frac{2(1-\sigma_G)}{\Lambda_\pi \kappa \lambda} \left(\mu_{2,t} - \mu_{2,t-1}\right), \\
&\quad - \frac{4(1-\sigma_G)\kappa\varphi}{\Lambda_\pi \kappa \lambda (1+\delta+\kappa)} \left(\mu_{5,t} - \mu_{5,t-1}\right), \\
\mu_{5,t} &= (1-\gamma)\beta_R \upsilon \mu_{2,t} + \frac{1}{1+\delta+\kappa}\mu_{5,t-1},
\end{aligned} \tag{24}$$

where $\mu_{5,t}$ denotes the Lagrange multiplier associated with Eq.(16), the NKRD. The two equalities in Eq.(24) imply not only that the inflation–output tradeoffs do not disappear simultaneously, but also that the relationship between inflation and the output gap is weakened.

4.1.2 Optimal Monetary Policy and Fiscal Policy

Under the optimal monetary and fiscal policy, the FONC for union-wide inflation and the output gap is given by Eq.(22). Thus, the union-wide inflation and the welfare -relevant output gap are stabilized by optimal monetary policy and fiscal policy although nontradables exist in a currency union.

The FONC for relative block inflation and the welfare-relevant output gap is given by:

$$\pi^R_{P,t} = -\frac{\Lambda_y (1-\sigma_G)}{\Lambda_\pi \kappa \lambda} \left(\hat{y}^R_t - \hat{y}^R_{t-1}\right). \tag{25}$$

This equality also implies that relative inflation and the welfare-relevant output gap are stabilized by optimal monetary policy and fiscal policy. Both Eqs.(22) and (25) imply that inflation–output tradeoffs disappear, not only at the union-wide level, but also in each country under the optimal monetary and fiscal policy regime. Note that we have Eq.(25), even though nontradables exist.

Why do we have Eq.(25) rather than Eq.(24)? Under the optimal monetary and fiscal policy, we have $\mu_{2,t} = 0$ as the optimality condition along with optimality conditions derived under the optimal monetary policy alone, because we not only have the nominal interest rate, but also government bonds in the policy function. We obtain Eq.(25) by substituting $\mu_{2,t} = 0$ along with the initial condition $\mu_{5,-1} = 0$ into Eq.(24). Because of this, we have $\mu_{5,t} = 0$, although this equality is not obtained directly by implementation of fiscal policy. This fact implies that optimal fiscal policy removes the effects of the CPI disparity, which introduces the consumption disparity between both countries by removing the disparity in the demand block.

4.2 Sensitivity Analysis

4.2.1 Parameterization

We run a series of dynamic simulations and adopt the following benchmark parameterization. We set the values of the inverse of the labor supply elasticity φ, the elasticity of substitution across goods θ, the elasticity of substitution between tradables and nontradables η, the subjective discount factor δ, the steady-state share of government bonds to output σ_B, the steady-state share of government expenditure to output σ_G and the tax rate τ equal to 3, 11, 0.75, 0.5, 1, 0.99, 2.4, 0.276 and 0.3, respectively, which is consistent with quarterly time periods in the model.[18] Except for α, γ, φ and η, these parameterizations are used in Ferrero[16].[19] According to our calculation, nontradables account for 50.3% of all goods in the Euro area; thus, we set the share of nontradables in the CPI as $\gamma = 0.5$.[20] Following Obstfeld and Rogoff[26], we set $\eta = 1$. We also assume that the government expenditure and productivity shifters are described according to the following AR(1) processes:

$$a_{H,t} = \rho_{\mathcal{T}} a_{H,t-1} + \xi_{H,t} \ ; \ a_{F,t} = \rho_{\mathcal{T}} a_{F,t-1} + \xi_{F,t}, \ ; \ a_{\mathcal{N},t} = \rho_{\mathcal{N}} a_{\mathcal{N},t-1} + \xi_{\mathcal{N},t}$$
$$a^*_{\mathcal{N},t} = \rho_{\mathcal{N}} a^*_{\mathcal{N},t-1} + \xi^*_{\mathcal{N},t} \ ; \ g^W_t = \rho_G g^W_{t-1} + \xi^W_{G,t} \ ; \ g^R_t = \rho_G g^R_{t-1} + \xi^R_{G,t},$$

where $\xi_{H,t}$, $\xi_{F,t}$, $\xi_{\mathcal{N},t}$, $\xi^*_{\mathcal{N},t}$, $\xi^W_{G,t}$ and $\xi^R_{G,t}$ denote the i.i.d. shocks. We set $\rho_{\mathcal{T}}$, $\rho_{\mathcal{N}}$ and ρ_G equal to 0.705, 0.784 and 0.8, following Batini, Harrison and Millard[2] and Ribeiro[28].[21]

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[18] $\sigma_B = 2.4$ implies that the steady-state debt–output annual ratio is 0.6.

[19] Many DSGE studies use the parameter values in Rotemberg and Woodford[29]. However, to compare our results with those derived by Ferrero[16] and to analyze the Euro area, we mainly use his parameter values for the Euro area. More precisely, the parameter values of θ, δ, σ_B, σ_G are set as in Ferrero[16]. Ferrero[16], however, sets a different degree of price rigidity in countries H and F. We set α equal to 0.75, which is assumed by Beetsma and Jensen[3]. Because φ does not appear in Ferrero[16], we set it equal to 3, which is adopted by Gali and Monacelli[18].

[20] Following the definition that regards goods produced in the manufacturing industry, agriculture, forestry, fishery and mining as tradables and regards goods produced in other industries as nontradables, as used by Canzoneri, Cumby and Diba[11], nontradables, in terms of both current and purchaser's prices, accounted for 50.3% of the sum of nontradables and tradables in major Euro area countries such as Belgium, Germany, France, Greece, Italy, the Netherlands, Portugal and Spain in 1999.

[21] There are few papers that estimate AR(1) parameters associated with the productivity of the tradables and nontradables sectors separately. Following Benigno and Thoenissen[8], we adopt the result of Batini, Harrison and Millard[2], who estimate AR(1) parameters associated with the productivity of tradables and nontradables sectors separately. Note that we recognize

To examine the impulse response functions (IRFs), we consider one percent changes in the productivity shifter of tradables in country H, $a_{H,t}$, and the productivity shifter of nontradables in country H, $a_{\mathcal{N},t}$, to investigate the effects of the existence of nontradables. Additionally, to compare our results with those of Ferrero[16], we consider one percent changes in the union-wide government expenditure shifter, g_t^W, and the relative government expenditure shifter, g_t^R.

4.2.2 Optimal Monetary Policy Alone

Figures 1 and 2 depict the IRFs under optimal monetary policy alone with commitment, in the case where all goods are tradable, namely $\gamma = 1$, and the benchmark case, namely $\gamma = 0.5$, respectively.[22] First, we consider changes in the productivity shifter of tradables in country H. In both cases, an increase in the productivity shifter of tradables in country H causes a decrease in the PPI inflation rate in country H through a decrease in the marginal cost of tradables in country H (17th panel in Figures 1 and 2). When all goods are tradables, this change simply decreases the welfare-relevant output gap in country H (9th panel in Figure 1). Furthermore, when half of the goods are tradables, a decrease in the marginal cost of tradables in country H stemming from an increase in the productivity of tradables produced in country H causes a decrease in the marginal cost of nontradables produced in country H. This is the cause of a relative decrease in the CPI in country H (29th panel in Figure 2). This is in contrast with the case where all goods are tradables where the CPI disparity does not change (29th panel in Figure 1). A relative decrease in the CPI in country H causes an increase in the NPD. A decrease in the price of nontradables in country H boosts demand for nontradables in country H. Thus, the decrease in the welfare-relevant output gap in country H in the benchmark case is smaller than that in the case where all goods are tradables (9th panel in Figures 1 and 2). This increase in the CPI disparity is inconsistent with the Balassa–Samuelson theorem, which implies a decrease in the CPI disparity. This increase in the CPI disparity is consistent with Benigno and Thoenissen[8]'s real exchange rate anomally.

The volatility of the PPI inflation rate to changes in the productivity of tradables in country H under the case where all goods are tradables and the benchmark case, are 0.0080 and 0.0081, respectively (cells in the 11th row and 3rd column in Tables 1 and 2). Intuitively, the volatility of the PPI inflation rate in the benchmark case is smaller than that in the case where all goods are tradables, because just half of the goods are tradable. In the benchmark case, however, a decrease in the marginal cost in the tradables sector decreases the marginal cost in the nontradables sector. Thus, the effect of the changes

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that their estimated parameters are smaller than those used in most RBC studies. Ribeiro[28] is one of the few papers that estimate autoregressive processes of government expenditure in Europe. We adopt his estimation result.

[22] Given the benchmark parameterization, the four eigenvalues are larger than 1 in value for the four forward-looking variables under the optimal monetary policy alone regime. The Blanchard–Kahn conditions are met.

in the productivity of tradables in country H to the PPI inflation rate is larger in the benchmark case. Clearly, existence of nontradables makes the PPI inflation rate more volatile. In addition, as mentioned in the latter section, this decreases social welfare and makes fiscal policy more important when there are nontradables.

The dynamics brought about by changes in the productivity shifter of tradables in country H can be confirmed by investigating the model, especially the relative block. Eq.(14) implies that an increase in the productivity shifter of tradables in country H decreases the welfare-relevant output gap disparity between both countries. The definition of NPD inflation and Eq.(16) shows that a decrease in the welfare-relevant output gap disparity between both countries increases the CPI disparity although a decrease in the welfare-relevant output gap disparity does not change the CPI disparity in the case where all goods are tradables. Monetary policy alone can simultaneously stabilize both the welfare-relevant output gap and the union-wide inflation rate by increasing the nominal interest rate.[23] However, the country-level welfare-relevant output gap and inflation rate cannot simultaneously be stabilized in both cases. In the case where all goods are tradables, a distorted steady state avoids simultaneous stabilization of both the welfare-relevant output gap and the inflation rate, as shown in Eq.(14). In the benchmark case, both a distorted steady state and the existence of nontradables avoids simultaneous stabilization of both the welfare-relevant output gap and the inflation rate, as shown in Eqs.(14) and (16). Because these two equalities help avoid simultaneous stabilization, fluctuations of PPI inflation in the benchmark case are larger than that in the case where all goods are tradables.

Regarding the benchmark case, we can explain the result of changes in the productivity shifter of nontradables in country H in the same manner as we can explain changes in the productivity shifter of tradables in country H. However, the coefficient of the productivity shifter of nontradables in the NKRD Eq.(16) is larger than the coefficient of the productivity shifter of tradables. Thus, the CPI disparity increases and the volatility of other macroeconomic variables is higher following changes in the productivity of nontradables than following changes in that of tradables (30th panel in Figure 2 and 7th, 9th, 11th, 13th, 15th and 17th rows in Table 2). As mentioned, an increase in the CPI disparity is consistent with Lipinska's (2008) finding that nontradables' productivity shocks lead to a stronger real exchange rate depreciation. In the case where all goods are tradables, any macroeconomic variables do not fluctuate when the productivity shifter of nontradables in country H changes because only tradables exist.

Now, we consider the innovation in union-wide government expenditure. Both IRFs and macroeconomic volatilities are the same in both cases because the union-wide government expenditure does not affect the relative block of the model. To secure funds for government expenditure, the nominal interest rate decreases and there is pressure for the welfare-relevant output gap to increase

[23] Both the union-wide inflation rate and the union-wide welfare-relevant output gap deviate slightly from their steady state, because of Eq.(21). See footnote 17 for a quantitative measure.

(27th panel in Figures 1 and 2). As implied in Eq.(13), however, a decrease in the lagged nominal interest rate decreases the union-wide welfare-relevant output gap. Thus, the union-wide welfare-relevant output gap is stabilized, although it increases at first (3rd panel in Figures 1 and 2).[24] The union-wide inflation rate is stabilized despite an increase in the union-wide welfare-relevant output gap, because an increase in government expenditure decreases the PPI inflation rate through Eq.(21) (7th panel in Figures 1 and 2). Note that because of Eq.(21), inflation–output gap tradeoffs are no longer dissolved completely, even though the optimality condition in Eq.(22) is applied when union-wide government expenditure increases. Along with the union-wide welfare-relevant output gap, the welfare-relevant output gap in each country is stabilized (11th and 15th panels in Figures 1 and 2). Furthermore, the PPI inflation rate in countries H and F is stabilized (19th and 23rd panels in Figures 1 and 2). Note that Ferrero[16] shows that the nominal interest rate increases to a unit innovation in union-wide government expenditure because the tax rate increases simultaneously to secure funds to finance government expenditure, whereas the tax rate is constant over time in our setting.

When relative government expenditure changes, the welfare-relevant output gap disparity between countries and the welfare-relevant output gap and inflation rate cannot be stabilized at the country level in both cases (12th, 16th, 20th and 24th panels in Figures 1 and 2).

Benigno[5], who assumes all goods are tradable, implies that monetary policy alone can simultaneously stabilize both the output gap and the inflation rate in each country even though asymmetric shocks hit the economy. In our model, when asymmetric shocks hit the economy, the PPI inflation rate and welfare-relevant output gap in each country are no longer stabilized regardless of the share of tradables γ. This fact stems from the distorted steady state, which is assumed by both us and Ferrero[16]. In addition, the volatilities of the PPI inflation rate in response to each shock in the benchmark case are larger than those in the case where all goods are tradables, except for the volatility of the PPI inflation rate to the union-wide government expenditure shock (11th and 13th rows in Tables 1 and 2). This fact reflects that nontradables amplifies the effects of asymmetric shocks.

4.2.3 Optimal Monetary Policy and Fiscal Policy Mix

Figures 3 and 4 depict the IRFs under the optimal monetary policy and fiscal policy mix with commitment in the cases where all goods are tradables and the benchmark, respectively.[25] To eliminate the effects of changes in the productivity shifter of tradables in country H, the fiscal authorities in both countries

[24] As mentioned in Section 2.3, all interest-bearing assets held by households are in the form of government bonds. Lowering the interest rate in the previous period reduces the nominal payoff of the bond portfolio purchased by households. Because of the budget constraint, this reduces output via a reduction in consumption.

[25] Given the benchmark parameterization, five eigenvalues are larger than 1 for the five forward-looking variables under the optimal monetary and fiscal policy regime. Thus, the Blanchard–Kahn conditions are met.

increase government bonds in both cases (33rd and 37th panels in Figures 3 and 4). An increase in government bonds during the previous period increases the nominal payoff of the bond portfolio purchased by households. Thus, consumption in both countries increases. Because an increase in government bonds in country H is higher than that in country F, an increase in consumption is higher than that in country F. This eliminates the pressure to decrease the welfare-relevant output gap in country H and eliminates the pressure to increase the welfare-relevant output gap in country F. Thus, the welfare-relevant output gap in both countries is stabilized. Under the optimal monetary policy and fiscal policy mix, the volatility of the welfare-relevant output gap in both countries dramatically decreases compared with the volatility under monetary policy alone in both cases (7th to 10th rows in Tables 1 and 2). Because the welfare-relevant output gap is well stabilized, the PPI inflation rate in each country is also well stabilized in both cases (17th and 21st panels in Figures 3 and 4). The volatility of the PPI inflation rate in both countries decreases dramatically compared with the volatility under monetary policy alone in both cases (cells in the 11th to 14th rows and 3rd column in Tables 1 and 2). In the case where all goods are tradables, CPI disparity does not fluctuate because all goods are tradables, which means that PPP applies (29th panel in Figure 3 and the cell at the 16th row and 3rd column in Table 1). In the benchmark case, because the PPI inflation rate in country H increases only in the first period, the CPI disparity decreases (29th panel in Figure 4). However, the volatility of the CPI disparity decreases dramatically compared with the volatility under monetary policy alone (cells in the 15th and 16th rows and 3rd column in Table 2).

Now, we consider changes in the productivity shifter of nontradables. Here the macroeconomic variables do not fluctuate in response to changes in the productivity of nontradables in the case where all goods are tradables. Thus, we focus only on macroeconomic volatility and fluctuations in the benchmark case. An increase in the quantity of government bonds in the previous period drastically stabilizes the welfare-relevant output gap (10th and 14th panels in Figure 4). Because of this, the PPI inflation also stabilizes (18th and 22nd panels in Figure 4). The volatility of the PPI inflation rate and the welfare-relevant output gap is less than that under optimal monetary policy alone (cells in the 7th to 14th rows and 4th column in Table 2). As shown in Eq.(17), the CPI disparity decreases (30th panel in Figure 4). The CPI disaprity is dramatically stabilized compared with that under the optimal monetary policy alone (cells in the 15th and 16th rows and 4th column in Table 2).

Next, we consider changes in union-wide government expenditure. Apart from government bonds, the mechanism of the macroeconomic variables' behavior is similar to that under optimal monetary policy alone in both cases. Although both fiscal authorities affiliate with the central bank, the IRFs and the macroeconomic volatilities behave the same as those under optimal monetary policy alone regarding the welfare-relevant output gap, the PPI inflation rate and the CPI disparity (3rd, 7th, 11th, 15th, 19th and 23rd panels in Figures 3 and 4 and cells in the 3rd to 16th rows and 5th column in Tables 1 and 2) in each case. This implies that there is no notable role for fiscal policy, which could replace monetary policy to stabilize the economy. When a union-wide shock, such as union-wide government expenditure, hits the economy, however, the central bank and the fiscal authorities cooperate against a union-wide shock. This result is the same as the one derived by Ferrero[16].

When the relative government expenditure shifter shocks the economy, the fiscal authority in country H decreases the quantity of government bonds while the fiscal authority in country F increases the quantity of government bonds in both cases (36th and 40th panels in Figures 3 and 4). As Eq.(14) implies, an increase in relative government expenditure increases the welfare-relevant output gap disparity. However, a decrease in the quantity of government bonds relative to the previous period decreases the current welfare-relevant output gap disparity. This eliminates the pressure to increase the PPI inflation rate in country H and to decrease it in country F. Thus, the PPI inflation rates in both countries are stabilized immediately in both cases (20th and 24th panels in Figures 3 and 4). The volatility of the PPI inflation rate is dramatically reduced in both cases (cells in the 11th to 14th rows and 6th column in Tables 1 and 2).

The role of the fiscal authorities is larger than that of the central bank in stabilizing both inflation and the welfare-relevant output gap in both cases. This fact strongly supports the implication derived by Ferrero[16], who insists on the importance of optimal monetary and fiscal policy in a currency union. In addition, we insist that the role of optimal fiscal policy as a stabilization tool when there are nontradables is more important than that when there are only tradables, which has already been mentioned above. The volatility of the PPI inflation rate decreases from 0.0121 to 1.4184e-005 in country H and 2.2170e-006 in country F, and from 0.0408 to 3.6616e-004 in the benchmark case, by adding optimal fiscal policy for changes in the productivity of nontradables and in the changes in the relative government expenditures (cells in the 11th to 14th rows and 4th and 6th columns in Table 2). In contrast, the volatility of the PPI inflation rate is unchanged at zero and falls from 0.0301 to 3.6616e-004 in the case where all goods are tradables, for changes in the productivity of nontradables and in changes in relative government expenditure, respectively (cells in the 11th to 14th rows and 4th and 6th columns in Table 1). Clearly, the degree of improvement in the volatility in the benchmark case is larger than that in the case where all goods are tradables. A more formal discussion of this from the viewpoint of welfare appears in Section 4.3.

Behind the stabilization of the volatility of the PPI inflation rate in the benchmark case for changes in productivity, there is stabilization in the CPI disparity. By comparing the 29th and 30th panels in Figure 2 with those in Figure 4, we confirm that the CPI disparity is stabilized. In fact, the volatility of the CPI disparity for changes in the productivity in the benchmark case under the optimal monetary and fiscal policy are smaller than that under the optimal monetary policy alone and are close to zero, which is the volatility for optimal monetary policy alone in the case where all goods are tradables (cells in the 15th and 16th rows and 3rd and 4th columns in Tables 1 and 2). Optimal fiscal policy stabilizes the PPI inflation rate by stabilizing the CPI disparity: namely, minimizing the consumption disparity between the two countries.

Finally, we point out that the volatility of the nominal interest rate decreases under the optimal monetary policy and fiscal policy. Furthermore, this tendency does not depend on the share of tradables (17th and 18th rows in Tables 1 and 2).

The results of this sensitivity analysis prove the policy implications that are mentioned by McKinnon[24]. Cooperative fiscal authorities have a certain role in stabilizing both the welfare-relevant output gap and inflation simultaneously when nontradables exist.

4.3 Welfare Analysis

In this section, we analyze the social welfare associated with both regimes, focusing on the share of nontradables. This paper finds that the role of optimal fiscal policy under the assumption that there are nontradables is more important than the one under the assumption that all goods are tradable. To clarify the role of optimal fiscal policy, we compare our result to that of Ferrero[16], who finds that such a policy is essential for the minimization of social losses when all goods are tradable.

Now, we define the welfare criteria. Setting $\delta \to 1$ on Eq.(19), the expected welfare losses of any policy can be written in terms of the variances of inflation and the welfare-relevant output gap as follows:

$$\tilde{\mathcal{L}}^W \equiv \frac{\Lambda_\pi}{4} \mathrm{var}\left(\pi_{P,t}\right) + \frac{\Lambda_\pi}{4} \mathrm{var}\left(\pi^*_{P,t}\right) + \frac{\Lambda_y}{4} \mathrm{var}\left(\hat{y}_t\right) + \frac{\Lambda_y}{4} \mathrm{var}\left(\hat{y}^*_t\right),$$

with $\tilde{\mathcal{L}}^W$ being the welfare loss under the setting $\delta \to 1$.

Figure 5 depicts social losses associated with the two regimes analyzed in the previous section: optimal monetary policy alone and optimal monetary policy and fiscal policy. As noted above, both regimes are fully committed. Under optimal monetary policy alone, when the share of nontradables increases, welfare losses increase. However, optimal monetary and fiscal policy bring about approximately zero welfare losses, independent of the share of nontradables.[26] The necessity of optimal fiscal policy is clear from this analysis. Note that optimal monetary policy alone cannot result in a zero welfare loss when all goods are tradable, while the welfare losses are minimized among the losses brought about by the optimal monetary policy alone. As Benigno[5] mentions, when all goods are tradable, optimal monetary policy alone can eliminate the inflation–output tradeoffs approximately and simultaneously.[27] However, Ferrero[16] insists that fiscal policy is needed, even though all goods are tradable. This discrepancy stems from the assumption of the steady-state behavior of the fiscal authority. Using DSGE analysis, Benigno[5] assumes a zero steady-state value of government expenditure and the quantity of bonds. As in our setting, Ferrero[16] does

[26] Because of Eq.(21), zero welfare losses definitely cannot be obtained although optimal fiscal policy is conducted.

[27] In addition, it is attained when price stickiness is the same in both countries.

not assume a zero steady-state value of government expenditure and bonds. Government expenditure and bonds have nonzero values in the steady state. This results in additional elasticities, the steady-state share of government expenditure with respect to output, and the steady-state share of government bonds with respect to output, σ_G and σ_B, respectively. These elasticities change the format of the demand block of the economy, which inhibits perfect risk sharing. Thus, the study of Ferrero[16] corresponds with our study in the case where all goods are tradable.

Our results insist that the existence of nontradables creates acute losses in the Euro economy, not only because of the assumption of a steady state, but also because of the real exchange rate anomaly. In our benchmark setting where $\gamma = 0.5$, the welfare loss—the percentage deviation of utility from its steady state, brought about by optimal monetary policy alone—is 0.26%, while it is 0.13% when all goods are tradable, $\gamma = 1$. As noted above, approximately 50.3% of goods are nontradable; thus, the role of optimal fiscal policy is greater than that suggested by Ferrero[16].

5 Implementing a Cooperative Solution by Self-oriented Fiscal Authorities

Some studies, such as Benigno[4], Obstfeld and Rogoff[27] and Benigno and Benigno[7], show that self-oriented monetary authorities can replicate the cooperative outcome in a decentralized framework so that there is no need for international monetary policy cooperation. Following their context, we investigate whether it is possible that fiscal policies set in a noncooperative environment can implement the optimal cooperative solution in this section.

While the central bank commits to minimizing the union-wide social loss $\mathcal{L}^W$ subject to the structural model, we assume that each fiscal authority commits to minimizing its respective losses as follows:

$$\mathcal{L}^{NC} \equiv \mathrm{E}_0 \sum_{t=0}^{\infty} \delta^t L_t^{NC} \ ; \ \mathcal{L}^{NC*} \equiv \mathrm{E}_0 \sum_{t=0}^{\infty} \delta^t L_t^{NC*},$$

subject to the structural model with:

$$L_t^{NC} \equiv \frac{\Lambda_\pi}{2}\pi_{P,t}^2 + \frac{\Lambda_y}{2}\hat{y}_t^2 \ ; \ L_t^{NC*} \equiv \frac{\Lambda_\pi}{2}\left(\pi_{P,t}^*\right)^2 + \frac{\Lambda_y}{2}\left(\hat{y}_t^*\right)^2, \tag{26}$$

where L_t^{NC} denotes the social loss assigned to the fiscal authority in country H under the self-oriented setting.[28]

Next, we calculate the union-wide social loss under the cooperative setting $\mathcal{L}^W$, and the union-wide social loss brought about by self-oriented fiscal authorities in both countries $\mathcal{L}^{NCW} \equiv \frac{1}{2}\left[\mathcal{L}^{NC} + \mathcal{L}^{NC*}\right]$. For simplicity without loss of

[28] Following Beetsma and Jensen[3], we split the per period union-wide social loss function Eq.(20) as follows: $L_t^W = \frac{1}{2}\left(L_t + L_t^*\right)$ with $L_t \equiv \frac{\Lambda_\pi}{2}\pi_{P,t}^2 + \frac{\Lambda_y}{2}\hat{y}_t^2$ and $L_t^* \equiv \frac{\Lambda_\pi}{2}\left(\pi_{P,t}^*\right)^2 + \frac{\Lambda_y}{2}\left(\hat{y}_t^*\right)^2$.

generality, we assume $\rho_{\mathcal{T}} = \rho_{\mathcal{N}} = \rho_G = 0$ and each shock has constant variance. After tedious calculations, we have the following:

$$\begin{aligned}
\mathcal{L}^W &= \mathcal{L}^{NCW} \\
&= \frac{\Psi_1}{(1-\delta)\,2\left(1-\Psi_1^2\right)}\left[\frac{\Lambda_y \Omega_8 \Psi_1}{2} + \Lambda_\pi \left(1-\Psi_1\right)^2\right]\Big\{\left[\kappa(1+\varphi)\Omega_3\right]^2 \gamma^2 \left[\mathrm{var}\left(\xi_{H,t}\right)\right. \\
&\quad \left. + \mathrm{var}\left(\xi_{F,t}\right)\right] + \left[\kappa(1+\varphi)\Omega_3\right]^2 (1-\gamma)^2 \left[\mathrm{var}\left(\xi_{\mathcal{N},t}\right) + \mathrm{var}\left(\xi_{\mathcal{N},t}^*\right)\right] \\
&\quad + \left(\kappa\sigma_G\Omega_4\right)^2 \left[\mathrm{var}\left(\xi_{G,t}\right) + \mathrm{var}\left(\xi_{G,t}^*\right)\right]\Big\},
\end{aligned}$$

where $\Psi_1 < 1$ denotes one of the solutions to the characteristic equation of Eq.(21) and its counterpart in country F and Ω_8 denotes a complicated coefficient. This implies that a self-oriented fiscal authority can achieve the cooperative allocation in the Nash equilibrium without imposing a complicated loss function on fiscal authorities. That is to say, there are no gains from fiscal policy cooperation to maximize social welfare. Interestingly, $\mathcal{L}^W = \mathcal{L}^{NCW}$ is applied independent of the share of tradables. Our policy implication is applicable to both the cases where there either are nontradables or not. Thus, our policy implication is different from the result of not only Beetsma and Jensen[3] who show the necessity of fiscal policy cooperation in a currency union where all goods are tradable, but also by McKinnon[24] who insists on the necessity of moving fiscal policy control from the national government to the central government in a currency union with nontradables. Furthermore, our policy implication differs from Liu and Pappa[23], whose model is a flexible exchange rate two-country model with nontradables, and who highlight the importance of policy cooperation.

Benigno and Benigno[6][7] and Liu and Pappa[23] show the importance of policy cooperation by comparing the cooperative setting with the self-oriented setting, although they do not assume a currency union. Under special cases, however, Benigno and Benigno[6][7] and Liu and Pappa[23] imply that there is no or only a quantitatively small role for policy cooperation. Benigno and Benigno[6][7] show that when the inverse of relative risk aversion times the elasticity of substitution between domestic and foreign tradables is unity, the allocation brought about by self-oriented policy makers corresponds to the cooperative allocation. Under such a case, each policy authority can control their own output gap and inflation independently of the TOT. Liu and Pappa[23], whose model implies that the inverse of relative risk aversion, the elasticity of substitution between domestic tradables and foreign tradables and the elasticity of substitution between tradables and nontradables, are unity, show that the gains from policy cooperation are quantitatively small when two countries have symmetric trading structures, which implies that the ratio of tradables to the sum of tradables and nontradables are equal in the two countries. In such a case, the TOT externality disappears, because output gap and inflation are independent of the TOT, as mentioned by Benigno and Benigno[6][7]. Once the symmetric trade structure is applied in Liu and Pappa[23], their parameterization is consistent with Benigno and Benigno[6][7]'s special case and Liu and Pappa[23] show that there are small gains from policy cooperation in such a situation.

Our model implies that the inverse of relative risk aversion and the elasticity of substitution between domestic tradables and foreign tradables are unity, as in Liu and Pappa[23]. Furthermore, the share of tradables γ is equal in the two countries and the elasticity of substitution between tradables and nontradables η is assumed to be unity after Section 4. The assumption of $\eta = 1$ is also consistent with Liu and Pappa[23]'s setting. Thus, our settings correspond to Liu and Pappa[23]'s special case in which the gains from policy cooperation are quantitatively small. Although we should consider that Liu and Pappa[23] do not assume a currency union, our policy implication is not necessarily inconsistent with theirs.

It is important and interesting to compare our results with the policy implication derived by Beetsma and Jensen[3], who develop a two-country currency union model without nontradables, implying that fiscal policy cooperation is important. While the difference in the policy implication between us and Benigno and Benigno[6][7] and Liu and Pappa[23] depends on parameter values, the difference in the policy implication between us and Beetsma and Jensen[3] does not depend on parameter values. In fact, although Beetsma and Jensen[3] assume an Armington form of the consumption index, which consists of tradables produced in two countries, as do we, they do not assume a logarithmic utility function of consumption as in Eq.(1).[29] Beetsma and Jensen[3]'s parameterization is not consistent with Benigno and Benigno[6][7]'s special case in which the allocation brought about by self-oriented policy makers corresponds to the cooperative allocation. However, it cannot be said that the reason we have a different policy implication on fiscal policy cooperation results from our parameterization assumptions. Beetsma and Jensen[3] assume a household utility function in which government expenditure appears and fiscal authorities manage government expenditure, although we do not assume such a utility function, as shown in Eq.(1), and we assume that fiscal authorities manage government debt. By second-order approximating such a utility function including government expenditure, the cross-term of the logarithmic TOT and the relative government expenditure gap, which gives additional welfare benefits from the TOT externality, appears in their union-wide loss function. They set the relative risk aversion larger than unity in their benchmark case, which is not consistent with us. However, the coefficient of that cross-term does not depend on relative risk aversion and does not disappear if relative risk aversion becomes unity, which corresponds to our parameterization. This implies that whether the TOT externality appears or not depends on the stabilization tools used by the fiscal authorities because adopting the utility function including government expenditure makes it possible to solve an optimization problem for fiscal authorities that control government expenditure. They assume that the fiscal authorities' loss functions in a noncooperative setting are domestically oriented and do not include such a cross-term. In our setting, the union-wide loss function is sim-

[29]Our consumption index is given by Eq.(2). If there are only tradables, however, Eq.(2) reduces to a consumption index of tradables: namely, $C_t = 2C_{H,t}^{\frac{1}{2}} C_{F,t}^{\frac{1}{2}}$ is applied.

ply the weighted sum of each country's loss function, as shown in Eqs.(20) and (26) and there is no TOT externality. In Beetsma and Jensen[3]'s setting, however, the union-wide-level loss function is not simply the sum of each country's loss function, and there is a TOT externality, which gives each fiscal authority opportunity to change the TOT through government expenditure to enhance each country's social welfare. Clearly, policy cooperation is important in their setting.

As mentioned, the difference between the policy implication of Beetsma and Jensen[3] and ours does not depend on parameter values. That difference depends on the stabilization tools used by the fiscal authorities. Although further detailed discussion is needed, the difference in stabilization tools may affect the necessity or importance of policy cooperation. Inferring from a comparison between the results of Beetsma and Jensen[3] and us, managing government debt as a stabilization tool may be less costly than managing government expenditure, because managing government debt does not need policy cooperation which involves nonnegligible costs.[30]

6 Conclusion

We discussed optimal monetary and fiscal policy in a currency union. Our most important and interesting finding is that self-oriented fiscal policy can replicate the allocation brought about by a cooperative setting and there are no gains from fiscal policy cooperation, either when there are nontradables or when there are not. This policy implication contrasts with Beetsma and Jensen[3], who show the importance of policy cooperation. Taking into account the costs of policy cooperation, our main finding implies that cooperation on fiscal policy in the Euro zone may be futile, because there are no gains from policy cooperation. Our results imply that recent pressure for more fiscal policy cooperation in Europe is not necessarily beneficial.

Comparing our main finding with Beetsma and Jensen[3]'s finding, there is a possibility that the necessity or importance of policy cooperation depends on the choice of stabilization tools. The necessity or importance of cooperation has been discussed in preceding papers, focusing on the parameter values that results in a TOT externality. Beetsma and Jensen[3] assume that fiscal authorities manage government debt and they show the necessity of fiscal policy cooperation. As mentioned, the difference in the policy implication between this paper and Beetsma and Jensen[3] does not depend on parameter values, but rather depends on the stabilization tools used. Further discussion on the choice of stabilization tools to avoid welfare losses associated with policy cooperation is an important objective for future research.

[30] There are costs to forcing players to comply with the cooperation framework and to share their understanding of the game, according to Kawai[20].

A Details on Derivation of the Model

A.1 Households

Preferences of the representative household in countries H and F are given by:

$$\begin{aligned}\mathcal{U} &\equiv \mathrm{E}_0 \sum_{t=0}^{\infty} \delta^t \left(\ln C_t - \frac{1}{1+\varphi} N_t^{1+\varphi} \right), \\ \mathcal{U}^* &\equiv \mathrm{E}_0 \sum_{t=0}^{\infty} \delta^t \left(\ln C_t^* - \frac{1}{1+\varphi} \left(N_t^*\right)^{1+\varphi} \right),\end{aligned} \tag{A.1}$$

where C_t^* denotes consumption in country F, $N_t^* \equiv N_{F,t} + N_{\mathcal{N},t}^*$ denotes hours of work in country F, $N_{F,t} \equiv \int_1^2 N_{F,t}(f)\, df$ and $N_{\mathcal{N},t}^* \equiv \int_1^2 N_{\mathcal{N},t}^*(f)\, df$ denote hours of work to produce tradables produced in country F and nontradables produced in country F, respectively. The first equality in Eq.(A.1) is Eq.(1) in the text.

More precisely, private consumption is a composite index defined by:

$$\begin{aligned}C_t &\equiv \left[\gamma^{\frac{1}{\eta}} C_{\mathcal{T},t}^{\frac{\eta-1}{\eta}} + (1-\gamma)^{\frac{1}{\eta}} C_{\mathcal{N},t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, \\ C_t^* &\equiv \left[\gamma^{\frac{1}{\eta}} \left(C_{\mathcal{T},t}^*\right)^{\frac{\eta-1}{\eta}} + (1-\gamma)^{\frac{1}{\eta}} \left(C_{\mathcal{N},t}^*\right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}},\end{aligned} \tag{A.2}$$

with $C_{\mathcal{N},t} \equiv \left[\int_0^1 C_{\mathcal{N},t}(h)^{\frac{\theta-1}{\theta}} dh\right]^{\frac{\theta}{\theta-1}}$, $C_{\mathcal{N},t}^* \equiv \left[\int_1^2 C_{\mathcal{N},t}^*(f)^{\frac{\theta-1}{\theta}} dh\right]^{\frac{\theta}{\theta-1}}$ $C_{H,t} \equiv \left[\int_0^1 C_{H,t}(h)^{\frac{\theta-1}{\theta}} dh\right]^{\frac{\theta}{\theta-1}}$ and $C_{F,t} \equiv \left[\int_1^2 C_{F,t}(f)^{\frac{\theta-1}{\theta}} df\right]^{\frac{\theta}{\theta-1}}$, where the index $\{h, f\}$ denotes a variable that is specific to agents h and f, $C_{\mathcal{T},t}^*$ denotes the consumption index for tradables in country F, and $C_{\mathcal{N},t}^*$ denotes an index of consumption across the nontradable goods produced in country F. The first equality in Eq.(A.2) is Eq.(2) in the text.

A sequence of budget constraints is given by:

$$\begin{aligned}D_t^n + W_t N_t - S_t \geq{} & \int_0^1 P_{H,t}(h)\, C_{H,t}(h)\, dh + \int_1^2 P_{F,t}(f)\, C_{F,t}(f)\, df \\ & + \int_0^1 P_{\mathcal{N},t}(h)\, C_{\mathcal{N},t}(h)\, dh + \mathrm{E}_t Q_{t,t+1} D_{t+1}^n, \\ D_t^{n*} + W_t^* N_t^* - S_t^* \geq{} & \int_0^1 P_{H,t}(h)\, C_{H,t}^*(h)\, df + \int_1^2 P_{F,t}(f)\, C_{F,t}^*(f)\, df \\ & + \int_0^1 P_{\mathcal{N},t}^*(f)\, C_{\mathcal{N},t}^*(f)\, df + \mathrm{E}_t Q_{t,t+1} D_{t+1}^{n*},\end{aligned} \tag{A.3}$$

with $P_{H,t} \equiv \left[\int_0^1 P_{H,t}(h)^{1-\theta} dh\right]^{\frac{1}{1-\theta}}$, $P_{F,t} \equiv \left[\int_1^2 P_{F,t}(f)^{1-\theta} df\right]^{\frac{1}{1-\theta}}$ and $P_{\mathcal{N},t} \equiv \left[\int_0^1 P_{\mathcal{N},t}(h)^{1-\theta} dh\right]^{\frac{1}{1-\theta}}$, where $P_{\mathcal{N},t}^* \equiv \left[\int_1^2 P_{\mathcal{N},t}^*(f)^{1-\theta} df\right]^{\frac{1}{1-\theta}}$ denotes the price index of nontradables produced in country F and S_t^* denotes the lump sum taxes in country F.

The optimal allocation of any given expenditure within each category of goods yields the following demand functions:

$$C_{H,t}(h) = \left(\frac{P_{H,t}(h)}{P_{H,t}}\right)^{-\theta} C_{H,t}, \quad ; \quad C_{F,t}(f) = \left(\frac{P_{F,t}(f)}{P_{F,t}}\right)^{-\theta} C_{F,t},$$
$$C_{H,t}^*(h) = \left(\frac{P_{H,t}(h)}{P_{H,t}}\right)^{-\theta} C_{H,t}^*, \quad ; \quad C_{F,t}^*(f) = \left(\frac{P_{F,t}(f)}{P_{F,t}}\right)^{-\theta} C_{F,t}^*,$$
$$C_{\mathcal{N},t}(h) = \left(\frac{P_{\mathcal{N},t}(h)}{P_{\mathcal{N},t}}\right)^{-\theta} C_{\mathcal{N},t}, \quad ; \quad C_{\mathcal{N},t}^*(f) = \left(\frac{P_{\mathcal{N},t}^*(f)}{P_{\mathcal{N},t}^*}\right)^{-\theta} C_{\mathcal{N},t}^* \tag{A.4}$$

These equalities imply that $\int_0^1 P_{H,t}(h) C_{H,t}(h)\, dh = P_{H,t}C_{H,t}$, $\int_1^2 P_{F,t}(f) C_{F,t}(f)\, df = P_{F,t}C_{F,t}$, $\int_0^1 P_{\mathcal{N},t}(h) C_{\mathcal{N},t}(h)\, dh = P_{\mathcal{N},t}C_{\mathcal{N},t}$ and $\int_1^2 P_{\mathcal{N},t}^*(f) C_{\mathcal{N},t}^*(f)\, df = P_{\mathcal{N},t}^*C_{\mathcal{N},t}^*$.

Total consumption expenditures by households in countries H and F are given by:

$$\begin{aligned} P_{H,t}C_{H,t} + P_{F,t}C_{F,t} + P_{\mathcal{N},t}C_{\mathcal{N},t} &= P_tC_t, \\ P_{F,t}C_{F,t}^* + P_{H,t}C_{H,t}^* + P_{\mathcal{N},t}^*C_{\mathcal{N},t}^* &= P_t^*C_t^*. \end{aligned}$$

Combining Eq.(A.3) and these equalities, we obtain:

$$\begin{aligned} D_t^n + W_tN_t + S_t &\geq P_tC_t + \mathrm{E}_tQ_{t,t+1}D_{t+1}^n, \\ D_t^{n*} + W_t^*N_t^* + S_t^* &\geq P_t^*C_t^* + \mathrm{E}_tQ_{t,t+1}D_{t+1}^{n*}, \end{aligned} \tag{A.5}$$

where the first equality in Eq.(A.5) is Eq.(3) in the text.

Combining Eq.(A.4) and aggregators, we have:

$$C_{H,t} = \frac{1}{2}\left(\frac{P_{H,t}}{P_{\mathcal{T},t}}\right)^{-1} C_{\mathcal{T},t}, \quad ; \quad C_{F,t} = \frac{1}{2}\left(\frac{P_{F,t}}{P_{\mathcal{T},t}}\right)^{-1} C_{\mathcal{T},t},$$
$$C_{H,t}^* = \frac{1}{2}\left(\frac{P_{H,t}}{P_{\mathcal{T},t}}\right)^{-1} C_{\mathcal{T},t}^*, \quad ; \quad C_{F,t}^* = \frac{1}{2}\left(\frac{P_{F,t}}{P_{\mathcal{T},t}}\right)^{-1} C_{\mathcal{T},t}^*,$$
$$C_{\mathcal{T},t} = \gamma\left(\frac{P_{\mathcal{T},t}}{P_t}\right)^{-\eta} C_t, \quad ; \quad C_{\mathcal{N},t} = (1-\gamma)\left(\frac{P_{\mathcal{N},t}}{P_t}\right)^{-\eta} C_t,$$
$$C_{\mathcal{T},t}^* = \gamma\left(\frac{P_{\mathcal{T},t}}{P_t^*}\right)^{-\eta} C_t^*, \quad ; \quad C_{\mathcal{N},t}^* = (1-\gamma)\left(\frac{P_{\mathcal{N},t}^*}{P_t^*}\right)^{-\eta} C_t^*. \tag{A.6}$$

The first, second, fifth and sixth equalities in Eq.(A.6) are Eq.(5) in the text.

CPIs are given by:

$$\begin{aligned} P_t &\equiv \left[\gamma P_{\mathcal{T},t}^{1-\eta} + (1-\gamma) P_{\mathcal{N},t}^{1-\eta}\right]^{\frac{1}{1-\eta}}, \\ P_t^* &\equiv \left[\gamma P_{\mathcal{T},t}^{1-\eta} + (1-\gamma)\left(P_{\mathcal{N},t}^*\right)^{1-\eta}\right]^{\frac{1}{1-\eta}}, \end{aligned} \tag{A.7}$$

where P_t^* denotes the CPI in country F. The first equality in Eq.(A.7) is Eq.(4) in the text.

The representative household maximizes Eq.(A.1) subject to Eq.(A.5). Optimality conditions are given by:

$$\delta \mathrm{E}_t \left(\frac{C_{t+1}^{-1} P_t}{C_t^{-1} P_{t+1}} \right) = \frac{1}{R_t}, \quad ; \quad \delta \mathrm{E}_t \left[\frac{\left(C_{t+1}^*\right)^{-1} P_t^*}{\left(C_t^*\right)^{-1} P_{t+1}^*} \right] = \frac{1}{R_t}, \tag{A.8}$$

$$C_t N_t^{\varphi} = \frac{W_t}{P_t}, \quad ; \quad C_t^* \left(N_t^*\right)^{\varphi} = \frac{W_t^*}{P_t^*}. \tag{A.9}$$

The RHS of Eq.(A.8) is an intertemporal optimality condition in country F, whereas the RHS of Eq.(A.9) is an intratemporal optimality condition in country F. The LHS of both Eqs.(A.8) and (A.9) are Eq.(6) in the text.

Combining and iterating Eq.(A.8) with an initial condition, we have the following optimal risk-sharing condition:

$$C_t = \vartheta C_t^* \mathsf{Q}_t, \tag{A.10}$$

which is Eq.(7) in the text. When $C_{-1} = C_{-1}^* = P_{-1} = P_{-1}^* = 1$, we have $\vartheta = 1$.

A.2 Firms

Each producer can use a linear technology to produce a differentiated good as follows:

$$\begin{aligned} Y_{H,t}(h) = A_{H,t} N_{H,t}(h), \quad &; \quad Y_{\mathcal{N},t}(h) = A_{\mathcal{N},t} N_{\mathcal{N},t}(h), \\ Y_{F,t}(f) = A_{F,t} N_{F,t}(f), \quad &; \quad Y_{\mathcal{N},t}^*(f) = A_{\mathcal{N},t}^* N_{\mathcal{N},t}^*(f), \end{aligned} \tag{A.11}$$

with $Y_{H,t} \equiv \left(\int_0^1 Y_{H,t}(h)^{\frac{\theta-1}{\theta}} dh \right)^{\frac{\theta}{\theta-1}}$, $Y_{F,t} \equiv \left(\int_1^2 Y_{F,t}(f)^{\frac{\theta-1}{\theta}} df \right)^{\frac{\theta}{\theta-1}}$, $Y_{\mathcal{N},t} \equiv \left(\int_0^1 Y_{\mathcal{N},t}(h)^{\frac{\theta-1}{\theta}} dh \right)^{\frac{\theta}{\theta-1}}$ and $Y_{\mathcal{N},t}^* \equiv \left(\int_1^2 Y_{\mathcal{N},t}^*(f)^{\frac{\theta-1}{\theta}} df \right)^{\frac{\theta}{\theta-1}}$, where $A_{F,t}$ and $A_{\mathcal{N},t}^*$ denote stochastic productivity shifters associated with tradables and nontradables produced in country F, respectively. The first equalities in Eq.(A.11) are Eq.(8) in the text.

Using Dixit–Stiglitz aggregators, Eq.(A.11) can be rewritten as:

$$\begin{aligned} Y_{H,t} = \frac{A_{H,t} N_{H,t}}{\int_0^1 \frac{Y_{H,t}(h)}{Y_{H,t}} dh} \quad &; \quad Y_{\mathcal{N},t} = \frac{A_{\mathcal{N},t} N_{\mathcal{N},t}}{\int_0^1 \frac{Y_{\mathcal{N},t}(h)}{Y_{\mathcal{N},t}} dh}, \\ Y_{F,t} = \frac{A_{F,t} N_{F,t}}{\int_1^2 \frac{Y_{F,t}(f)}{Y_{F,t}} df} \quad &; \quad Y_{\mathcal{N},t}^* = \frac{A_{\mathcal{N},t}^* N_{\mathcal{N},t}^*}{\int_1^2 \frac{Y_{\mathcal{N},t}^*(f)}{Y_{\mathcal{N},t}^*} df}. \end{aligned} \tag{A.12}$$

Under Calvo–Yun-style price-setting behavior, the pricing rules are given by:

$$P_{H,t} = \left[\alpha P_{H,t-1}^{1-\theta} + (1-\alpha) \tilde{P}_{H,t}^{1-\theta} \right]^{\frac{1}{1-\theta}},$$

$$
\begin{aligned}
P_{\mathcal{N},t} &= \left[\alpha P_{N,t-1}^{1-\theta} + (1-\alpha)\,\tilde{P}_{\mathcal{N},t}^{1-\theta}\right]^{\frac{1}{1-\theta}}, \\
P_{F,t} &= \left[\alpha P_{F,t-1}^{1-\theta} + (1-\alpha)\,\tilde{P}_{F,t}^{1-\theta}\right]^{\frac{1}{1-\theta}}, \\
P_{\mathcal{N},t}^{*} &= \left[\alpha \left(P_{N,t-1}^{*}\right)^{1-\theta} + (1-\alpha)\left(\tilde{P}_{\mathcal{N},t}^{*}\right)^{1-\theta}\right]^{\frac{1}{1-\theta}},
\end{aligned}
\tag{A.13}
$$

where $\tilde{P}_{F,t}$ and $\tilde{P}_{\mathcal{N},t}^{*}$ are the prices chosen by firms when they obtain the chance to change prices associated with tradables and nontradables produced in country F, respectively.

The maximization problems faced by firms are as follows:

$$
\begin{aligned}
&\max_{\tilde{P}_{H,t}} \mathrm{E}_t \left[\sum_{k=0}^{\infty} (\alpha\delta)^k \left(P_{t+k}C_{t+k}\right)^{-1} \tilde{Y}_{H,t+k}\left(\tilde{P}_{H,t} - MC_{H,t+k}^{n}\right)\right], \\
&\max_{\tilde{P}_{\mathcal{N},t}} \mathrm{E}_t \left[\sum_{k=0}^{\infty} (\alpha\delta)^k \left(P_{t+k}C_{t+k}\right)^{-1} \tilde{Y}_{\mathcal{N},t+k}\left(\tilde{P}_{\mathcal{N},t} - MC_{\mathcal{N},t+k}^{n}\right)\right], \\
&\max_{\tilde{P}_{F,t}} \mathrm{E}_t \left[\sum_{k=0}^{\infty} (\alpha\delta)^k \left(P_{t+k}^{*}C_{t+k}^{*}\right)^{-1} \tilde{Y}_{F,t+k}\left(\tilde{P}_{F,t} - MC_{F,t+k}^{n}\right)\right], \\
&\max_{\tilde{P}_{\mathcal{N},t}^{*}} \mathrm{E}_t \left[\sum_{k=0}^{\infty} (\alpha\delta)^k \left(P_{t+k}^{*}C_{t+k}^{*}\right)^{-1} \tilde{Y}_{\mathcal{N},t+k}^{*}\left(\tilde{P}_{\mathcal{N},t}^{*} - MC_{\mathcal{N},t+k}^{*n}\right)\right],
\end{aligned}
$$

$\tilde{Y}_{H,t+k} \equiv \left(\frac{\tilde{P}_{H,t}}{P_{H,t+k}}\right)^{-\theta} Y_{H,t+k}$ and $\tilde{Y}_{\mathcal{N},t+k} \equiv \left(\frac{\tilde{P}_{\mathcal{N},t}}{P_{\mathcal{N},t+k}}\right)^{-\theta} Y_{\mathcal{N},t+k}$, where $\tilde{Y}_{F,t+k} \equiv \left(\frac{\tilde{P}_{F,t}}{P_{F,t+k}}\right)^{-\theta} Y_{F,t+k}$ and $\tilde{Y}_{\mathcal{N},t+k}^{*} \equiv \left(\frac{\tilde{P}_{\mathcal{N},t}^{*}}{Y_{\mathcal{N},t+k}^{*}}\right)^{-\theta} Y_{\mathcal{N},t+k}^{*}$ denote the total demands when the prices change, and $MC_{F,t}^{n} \equiv \frac{W_t^{*}}{(1-\tau)A_{F,t}}$ and $MC_{\mathcal{N},t}^{*n} \equiv \frac{W_t^{*}}{(1-\tau)A_{\mathcal{N},t}^{*}}$ denote the marginal costs associated with tradables and nontradables produced in country F, respectively.

The FONCs are as follows:

$$
\begin{aligned}
\mathrm{E}_t \left[\sum_{k=0}^{\infty} (\alpha\delta)^k \left(P_{t+k}C_{t+k}\right)^{-1} \tilde{Y}_{H,t+k}\left(\tilde{P}_{H,t} - \zeta MC_{H,t+k}^{n}\right)\right] &= 0, \\
\mathrm{E}_t \left[\sum_{k=0}^{\infty} (\alpha\delta)^k \left(P_{t+k}C_{t+k}\right)^{-1} \tilde{Y}_{\mathcal{N},t+k}\left(\tilde{P}_{\mathcal{N},t} - \zeta MC_{\mathcal{N},t+k}^{n}\right)\right] &= 0, \\
\mathrm{E}_t \left[\sum_{k=0}^{\infty} (\alpha\delta)^k \left(P_{t+k}^{*}C_{t+k}^{*}\right)^{-1} \tilde{Y}_{F,t+k}\left(\tilde{P}_{F,t} - \zeta MC_{F,t+k}^{n}\right)\right] &= 0, \\
\mathrm{E}_t \left[\sum_{k=0}^{\infty} (\alpha\delta)^k \left(P_{t+k}^{*}C_{t+k}^{*}\right)^{-1} \tilde{Y}_{\mathcal{N},t+k}^{*}\left(\tilde{P}_{\mathcal{N},t}^{*} - \zeta MC_{\mathcal{N},t+k}^{n*}\right)\right] &= 0.
\end{aligned}
\tag{A.14}
$$

The first and the second equalities in Eq.(A.14) are Eq.(9) in the text.

We define the real marginal costs as:

$$MC_{H,t} \equiv \frac{MC_{H,t}^n}{P_{P,t}} \quad ; \quad MC_{\mathcal{N},t} \equiv \frac{MC_{\mathcal{N},t}^n}{P_{P,t}},$$
$$MC_{F,t} \equiv \frac{MC_{F,t}^n}{P_{P,t}^*} \quad ; \quad MC_{\mathcal{N},t}^* \equiv \frac{(MC^*)_{\mathcal{N},t}^n}{P_{P,t}^*}, \tag{A.15}$$

with $P_P^* \equiv \frac{P_{F,t}Y_{F,t}+P_{\mathcal{N},t}^*Y_{\mathcal{N},t}^*}{Y_{F,t}+Y_{\mathcal{N},t}^*}$.

Combining the first equalities of Eqs.(A.14) and (A.15) yields:

$$\mathrm{E}_t\left\{\sum_{k=0}^{\infty}(\alpha\delta)^k\left[\tilde{\mathsf{X}}_{H,t+k}^{-(\theta-1)}\mathsf{X}_{T,t+k}^{-(\eta-1)} - \zeta\tilde{\mathsf{X}}_{H,t+k}^{-\theta}\mathsf{X}_{H,t+k}^{-1}\mathsf{X}_{\mathcal{T},t+k}^{-\eta}\mathsf{X}_{P,t+k}MC_{H,t+k}\right]\right\} = 0, \tag{A.16}$$

with $\tilde{\mathsf{X}}_{H,t+k} \equiv \frac{\tilde{P}_{H,t}}{P_{H,t+k}}$, $\mathsf{X}_{H,t+k} \equiv \frac{P_{H,t+k}}{P_{\mathcal{T},t+k}}$, $\mathsf{X}_{\mathcal{T},t+k} \equiv \frac{P_{\mathcal{T},t+k}}{P_{t+k}}$ and $\mathsf{X}_{P,t+k} \equiv \frac{P_{P,t+k}}{P_{t+k}}$.

Combining the definition of the marginal cost and Eq.(A.9), we have:

$$MC_{H,t} = \frac{C_t N_t^{\varphi} P_t}{(1-\tau)P_{P,t}A_{H,t}}, \quad ; \quad MC_{\mathcal{N},t} = \frac{C_t N_t^{\varphi} P_t}{(1-\tau)P_{P,t}A_{\mathcal{N},t}},$$
$$MC_{F,t} = \frac{C_t^*(N_t^*)^{\varphi} P_t^*}{(1-\tau)P_{P,t}^*A_{F,t}}, \quad ; \quad MC_{\mathcal{N},t}^* = \frac{C_t^*(N_t^*)^{\varphi} P_t^*}{(1-\tau)P_{P,t}^*A_{\mathcal{N},t}^*}. \tag{A.17}$$

We define the country-wide real marginal cost as:

$$MC_t \equiv \frac{MC_{H,t}Y_{H,t} + MC_{\mathcal{N},t}Y_{\mathcal{N},t}}{Y_{H,t}+Y_{\mathcal{N},t}},$$
$$MC_t^* \equiv \frac{MC_{F,t}Y_{F,t} + MC_{\mathcal{N},t}^*Y_{\mathcal{N},t}^*}{Y_{H,t}+Y_{\mathcal{N},t}}.$$

A.3 Local Government

The government expenditure index is given by:

$$G_{H,t} \equiv \left(\int_0^1 G_{H,t}(h)^{\frac{\theta-1}{\theta}}dh\right)^{\frac{\theta}{\theta-1}}, \quad ; \quad G_{\mathcal{N},t} \equiv \left(\int_0^1 G_{\mathcal{N},t}(h)^{\frac{\theta-1}{\theta}}df\right)^{\frac{\theta}{\theta-1}},$$
$$G_{F,t} \equiv \left(\int_1^2 G_{F,t}(f)^{\frac{\theta-1}{\theta}}dh\right)^{\frac{\theta}{\theta-1}}, \quad ; \quad G_{\mathcal{N},t}^* \equiv \left(\int_1^2 G_{\mathcal{N},t}^*(f)^{\frac{\theta-1}{\theta}}df\right)^{\frac{\theta}{\theta-1}},$$

where $G_{F,t}$ and $G_{\mathcal{N},t}^*$ denote government expenditure on tradables and non-tradables produced in country F, respectively. For simplicity, we assume that government purchases are fully allocated to a domestically produced good. For any given level of public consumption, the government allocates expenditures across goods in order to minimize total cost. This yields the following set of government demand schedules, analogous to those associated with private consumption.

$$G_{H,t}(h)=\left(\frac{P_{H,t}(h)}{P_{H,t}}\right)^{-\theta}G_{H,t} \quad ; \quad G_{\mathcal{N},t}(h)=\left(\frac{P_{\mathcal{N},t}(h)}{P_{\mathcal{N},t}}\right)^{-\theta}G_{\mathcal{N},t},$$
$$G_{F,t}(f)=\left(\frac{P_{F,t}(f)}{P_{F,t}}\right)^{-\theta}G_{F,t} \quad ; \quad G^{*}_{\mathcal{N},t}=\left(\frac{P^{*}_{\mathcal{N},t}(f)}{P^{*}_{\mathcal{N},t}}\right)^{-\theta}G^{*}_{\mathcal{N},t}. \quad \text{(A.18)}$$

The flow government budget constraints are given by:

$$\begin{aligned} B^{n}_{t} &= R_{t-1}B^{n}_{t-1}-\left\{\int_{0}^{1}P_{H,t}(h)\left[\tau Y_{H,t}(h)-G_{H,t}(h)\right]dh \right. \\ &\left. +\int_{0}^{1}P_{\mathcal{N},t}(h)\left[\tau Y_{\mathcal{N},t}(h)-G_{\mathcal{N},t}(h)\right]dh\right\} \\ B^{n*}_{t} &= R_{t-1}B^{n*}_{t-1}-\left\{\int_{1}^{2}P_{F,t}(h)\left[\tau Y_{F,t}(f)-G_{F,t}(h)\right]dh \right. \\ &\left. +\int_{1}^{2}P^{*}_{\mathcal{N},t}(h)\left[\tau Y^{*}_{\mathcal{N},t}(h)-G^{*}_{\mathcal{N},t}(h)\right]dh\right\} \end{aligned} \quad \text{(A.19)}$$

where $B^{n*}_{t} \equiv P^{*}_{t}B^{*}_{t}$ denote the nominal risk-free bonds issued by local government in country F and B^{*}_{t} denote the real risk-free bonds issued by local government in country F, respectively.

Combining the definition of prices and output, we have:

$$Y_{H,t}(h)=\left(\frac{P_{H,t}(h)}{P_{H,t}}\right)^{-\theta}Y_{H,t}, \quad ; \quad Y_{\mathcal{N},t}(h)=\left(\frac{P_{\mathcal{N},t}(h)}{P_{\mathcal{N},t}}\right)^{-\theta}Y_{\mathcal{N},t},$$
$$Y_{F,t}(f)=\left(\frac{P_{F,t}(f)}{P_{F,t}}\right)^{-\theta}Y_{F,t}, \quad ; \quad Y^{*}_{H,t}(h)=\left(\frac{P_{H,t}(h)}{P_{H,t}}\right)^{-\theta}Y^{*}_{H,t}. \text{(A.20)}$$

Substituting Eqs.(A.18) and (A.20) into Eq.(A.19), we have:

$$\begin{aligned} B^{n}_{t} &= R_{t-1}B^{n}_{t-1}-\left[\tau\left(P_{H,t}Y_{H,t}+P_{\mathcal{N},t}Y_{\mathcal{N},t}\right)-\left(P_{H,t}G_{H,t}+P_{\mathcal{N},t}G_{\mathcal{N},t}\right)\right], \\ B^{n*}_{t} &= R_{t-1}B^{n*}_{t-1}-\left[\tau\left(P_{F,t}Y_{F,t}+P^{*}_{\mathcal{N},t}Y^{*}_{\mathcal{N},t}\right)-\left(P_{F,t}G_{F,t}+P^{*}_{\mathcal{N},t}G^{*}_{\mathcal{N},t}\right)\right]. \end{aligned}$$

These equalities can be rewritten as:

$$\begin{aligned} B^{n}_{t} &= R_{t-1}B^{n}_{t-1}-\left[\tau P_{P,t}\left(Y_{H,t}+Y_{\mathcal{N},t}\right)-P_{G,t}\left(G_{H,t}+G_{\mathcal{N},t}\right)\right], \\ B^{n*}_{t} &= R_{t-1}B^{n*}_{t-1}-\left[\tau P^{*}_{P,t}\left(Y_{F,t}+Y^{*}_{\mathcal{N},t}\right)-P^{*}_{G,t}\left(G_{F,t}+G^{*}_{\mathcal{N},t}\right)\right], \end{aligned} \quad \text{(A.21)}$$

with $P^{*}_{G,t} \equiv \frac{P_{F,t}G_{F,t}+P^{*}_{\mathcal{N},t}G^{*}_{\mathcal{N},t}}{G_{F,t}+G^{*}_{\mathcal{N},t}}$. The first equality in Eq.(A.21) is Eq.(10) in the text.

Eq.(A.21) yields the consolidated government budget constraint which is given by:

$$\begin{aligned}\frac{1}{2}\left(B_t^n + B_t^{n*}\right) &= R_{t-1}\frac{1}{2}\left(B_{t-1}^n + B_{t-1}^{n*}\right) - \frac{1}{2}\left\{\left[\tau P_{P,t}\left(Y_{H,t} + Y_{\mathcal{N},t}\right) - P_{G,t}\left(G_{H,t} + G_{\mathcal{N},t}\right)\right],\right. \\ &\left. + \left[\tau P_{P,t}^*\left(Y_{F,t} + Y_{\mathcal{N},t}^*\right) - P_{G,t}^*\left(G_{F,t} + G_{\mathcal{N},t}^*\right)\right]\right\}. \qquad \text{(A.22)}\end{aligned}$$

The appropriate transversality conditions for government assets are given by:

$$\lim_{k\to\infty} \mathrm{E}_t Q_{t,k}\frac{1}{2}\left(B_k^n + B_k^{*n}\right) = 0,$$

which appears in footnote 9 in the text.

Starting from Eq.(A.22) with the appropriate transversality condition, the resulting consolidated intertemporal budget constraint can be written as:

$$\begin{aligned}&\frac{1}{2}R_{t-1}\left[\frac{C_t^{-1}}{\Pi_t}B_{t-1}\right. \\ &\left. + \frac{\left(C_t^*\right)^{-1}}{\Pi_t^*}B_{t-1}^*\right] = \frac{1}{2}\mathrm{E}_t\left\{\sum_{k=0}^{\infty}\delta^k\left[\frac{P_{P,t+k}\tau\left(Y_{H,t+k} + Y_{\mathcal{N},t+k}\right) - P_{G,t+k}\left(G_{H,t+k} + G_{\mathcal{N},t+k}\right)}{C_{t+k}P_{t+k}}\right.\right. \\ &\left.\left. + \frac{P_{P,t+k}^*\tau\left(Y_{F,t+k} + Y_{\mathcal{N},t+k}\right) - P_{G,t+k}^*\left(G_{F,t+k} + G_{\mathcal{N},t+k}^*\right)}{C_{t+k}^*P_{t+k}^*}\right]\right\},\end{aligned}$$

with $\Pi_t^* \equiv \frac{P_t^*}{P_{t-1}^*}$ being the gross CPI inflation rate in country F. This can be rewritten as:

$$\begin{aligned}\frac{1}{2}R_{t-1}\left[\frac{C_t^{-1}}{\Pi_t}B_{t-1} + \frac{\left(C_t^*\right)^{-1}}{\Pi_t^*}B_{t-1}^*\right] &= \frac{P_{P,t}\tau\left(Y_{H,t} + Y_{\mathcal{N},t}\right) - P_{G,t}\left(G_{H,t} + G_{\mathcal{N},t}\right)}{P_tC_t} \\ &+ \frac{P_{P,t}^*\tau\left(Y_{F,t} + Y_{\mathcal{N},t}^*\right) - P_{G,t}^*\left(G_{F,t} + G_{\mathcal{N},t}^*\right)}{P_t^*C_t^*} \\ &+ \delta\mathrm{E}_tR_t\left(\frac{C_{t+1}^{-1}}{\Pi_{t+1}}B_t + \frac{\left(C_{t+1}^*\right)^{-1}}{\Pi_{t+1}^*}B_t^*\right). \quad \text{(A.23)}\end{aligned}$$

A.4 Market Clearing

Market clearing conditions for tradables are given by:

$$\begin{aligned}Y_{H,t}(h) &= C_{H,t}(h) + C_{H,t}^*(h) + G_{H,t}(h), \\ Y_{F,t}(f) &= C_{F,t}(f) + C_{F,t}^*(f) + G_{F,t}(f). \qquad \text{(A.24)}\end{aligned}$$

The first equality in Eq.(A.24) is the LHS of Eq.(11) in the text.

As for nontradables, equilibrium requires that:

$$\begin{aligned} Y_{\mathcal{N},t}(h) &= C_{\mathcal{N},t}(h) + G_{\mathcal{N},t}(h), \\ Y^*_{\mathcal{N},t}(f) &= C^*_{\mathcal{N},t}(f) + G^*_{\mathcal{N},t}(f). \end{aligned} \tag{A.25}$$

The first equality in Eq.(A.25) is the RHS of Eq.(11) in the text.

The market in country H for tradables clears when domestic demand is given by Eq.(A.24). As for nontradables, equilibrium requires Eq.(A.25).

Using Eqs.(A.4), (A.10) and (A.18), Eq.(A.24) can be rewritten as:

$$\begin{aligned} Y_{H,t}(h) &= \left(\frac{P_{H,t}(h)}{P_{H,t}}\right)^{-\theta} \left\{\frac{\gamma}{2}\left(\frac{P_{H,t}}{P_{\mathcal{T},t}}\right)^{-1} C_t \left[\left(\frac{P_{\mathcal{T},t}}{P_t}\right)^{-\eta} + \left(\frac{P_{\mathcal{T},t}}{P_t^*}\right)^{-\eta} \mathsf{Q}_t^{-1}\right] + G_{H,t}\right\}, \\ Y_{F,t}(f) &= \left(\frac{P_{F,t}(f)}{P_{F,t}}\right)^{-\theta} \left\{\frac{\gamma}{2}\left(\frac{P_{F,t}}{P_{\mathcal{T},t}}\right)^{-1} C_t \left[\left(\frac{P_{\mathcal{T},t}}{P_t}\right)^{-\eta} + \left(\frac{P_{\mathcal{T},t}}{P_t^*}\right)^{-\eta} \mathsf{Q}_t^{-1}\right] + G_{F,t}\right\}, \end{aligned}$$

where we use the fact that $C_t^* = \frac{C_t}{\mathsf{Q}_t}$, which is derived from Eq.(A.10). Combining these equalities and Eqs.(A.4), (A.10) and (A.18), Eq.(A.24) can be rewritten as:

$$\begin{aligned} Y_{H,t} &= \frac{\gamma}{2}\left(\frac{P_{H,t}}{P_{\mathcal{T},t}}\right)^{-1} C_t \left[\left(\frac{P_{\mathcal{T},t}}{P_t}\right)^{-\eta} + \left(\frac{P_{\mathcal{T},t}}{P_t^*}\right)^{-\eta} \mathsf{Q}_t^{-1}\right] + G_{H,t}, \\ Y_{F,t} &= \frac{\gamma}{2}\left(\frac{P_{F,t}}{P_{\mathcal{T},t}}\right)^{-1} C_t \left[\left(\frac{P_{\mathcal{T},t}}{P_t}\right)^{-\eta} + \left(\frac{P_{\mathcal{T},t}}{P_t^*}\right)^{-\eta} \mathsf{Q}_t^{-1}\right] + G_{F,t}. \end{aligned} \tag{A.26}$$

Using Eqs.(A.4), (A.10) and (A.18), Eq.(A.25) can be rewritten as:

$$\begin{aligned} Y_{\mathcal{N},t}(h) &= \left(\frac{P_{\mathcal{N},t}(h)}{P_{\mathcal{N},t}}\right)^{-\theta} \left[(1-\gamma)\left(\frac{P_{\mathcal{N},t}}{P_t}\right)^{-\eta} C_t + G_{\mathcal{N},t}\right], \\ Y^*_{\mathcal{N},t}(f) &= \left(\frac{P_{\mathcal{N},t}(h)}{P_{\mathcal{N},t}}\right)^{-\theta} \left[(1-\gamma)\left(\frac{P^*_{\mathcal{N},t}}{P_t^*}\right)^{-\eta} C_t \mathsf{Q}_t^{-1} + G^*_{\mathcal{N},t}\right]. \end{aligned}$$

Combining these equalities and Eqs.(A.4), (A.10) and (A.18), Eq.(A.25) can be rewritten as:

$$\begin{aligned} Y_{\mathcal{N},t} &= (1-\gamma)\left(\frac{P_{\mathcal{N},t}}{P_t}\right)^{-\eta} C_t + G_{\mathcal{N},t}, \\ Y^*_{\mathcal{N},t} &= (1-\gamma)\left(\frac{P^*_{\mathcal{N},t}}{P_t^*}\right)^{-\eta} C_t \mathsf{Q}_t^{-1} + G^*_{\mathcal{N},t}. \end{aligned} \tag{A.27}$$

Eq.(A.26) implies that:

$$\frac{Y_{H,t} - G_{H,t}}{Y_{F,t} - G_{F,t}} = \mathsf{T}_t,$$

where $\mathsf{T}_t \equiv \frac{P_{F,t}}{P_{H,t}}$ denotes the terms of trade (TOT).

Eq.(A.27) implies that:

$$\frac{Y_{\mathcal{N},t} - G_{\mathcal{N},t}}{Y^*_{\mathcal{N},t} - G^*_{\mathcal{N},t}} = \mathsf{N}_t^{\eta} \mathsf{Q}_t^{-(\eta-1)},$$

where $\mathsf{N}_t \equiv \frac{P^*_{\mathcal{N},t}}{P_{\mathcal{N},t}}$ denotes the nontradables price difference between countries H and F (NPD).

Finally, we define country-wide output and government expenditure as:

$$Y_t \equiv \frac{P_{H,t}}{P_{P,t}} Y_{H,t} + \frac{P_{\mathcal{N},t}}{P_{P,t}} Y_{\mathcal{N},t} \quad ; \quad Y_t^* \equiv \frac{P_{F,t}}{P^*_{P,t}} Y_{F,t} + \frac{P_{\mathcal{N},t}}{P^*_{P,t}} Y_{\mathcal{N},t}, \tag{A.28}$$

$$G_t \equiv \frac{P_{H,t}}{P_{G,t}} G_{H,t} + \frac{P_{\mathcal{N},t}}{P_{G,t}} G_{\mathcal{N},t} \quad ; \quad G_t^* \equiv \frac{P_{F,t}}{P_{G,t}} G_{F,t} + \frac{P^*_{\mathcal{N},t}}{P^*_{G,t}} G_{\mathcal{N},t}. \tag{A.29}$$

The LHS equalities in Eqs.(A.28) and (A.29) are Eq.(12) in the text.

A.5 Net Exports

Following Gali and Monacelli[18], we define net exports in country H as follows:

$$NX_t \equiv Y_t - \frac{P_t}{P_{P,t}} C_t - \frac{P_{G,t}}{P_{P,t}} G_t, \tag{A.30}$$

where NX_t denotes net exports in country H.

B Nonstochastic Steady State

We focus on equilibria where the state variables follow paths that are close to a deterministic stationary equilibrium, in which $\Pi_{H,t} = \Pi_{\mathcal{N}} = \Pi_{F,t} = \Pi^*_{\mathcal{N},t} = 1$ with $\Pi_{H,t} \equiv \frac{P_{H,t}}{P_{H,t-1}}$, $\Pi_{\mathcal{N},t} \equiv \frac{P_{\mathcal{N},t}}{P_{\mathcal{N},t-1}}$, $\Pi_{F,t} \equiv \frac{P_{F,t}}{P_{F,t-1}}$ and $\Pi^*_{\mathcal{N},t} \equiv \frac{P^*_{\mathcal{N},t}}{P^*_{\mathcal{N},t-1}}$ where variables without the subscript indicating the period denote their nonstochastic steady state value. These imply that the PPI inflation rate is zero in this steady state. Note that $\tilde{\mathsf{X}}_H = \tilde{\mathsf{X}}_{\mathcal{N}} = \tilde{\mathsf{X}}_F = \tilde{\mathsf{X}}^*_{\mathcal{N}} = 1$ is applied in this steady state with $\tilde{\mathsf{X}}_{H,t} \equiv \frac{\tilde{P}_{H,t}}{P_{H,t}}$, $\tilde{\mathsf{X}}_{\mathcal{N},t} \equiv \frac{\tilde{P}_{\mathcal{N},t}}{P_{\mathcal{N},t}}$, $\tilde{\mathsf{X}}_{F,t} \equiv \frac{\tilde{P}_{F,t}}{P_{F,t}}$ and $\tilde{\mathsf{X}}^*_{\mathcal{N},t} \equiv \frac{\tilde{P}^*_{\mathcal{N},t}}{P^*_{\mathcal{N},t}}$. Because this steady state is nonstochastic, all productivities are unit values, i.e., $A_H = A_{\mathcal{N}} = A_F = A^*_{\mathcal{N}} = 1$. In addition, we assume that $G_H = G_F$, $G_{\mathcal{N}} = G^*_{\mathcal{N}}$ and $B = B^*$ in this steady state.

In this steady state, the gross nominal interest rate is equal to the inverse of the subjective discount factor, as follows:

$$R = \delta^{-1}.$$

Eq.(A.14) can be rewritten as:

$$\tilde{P}_{H,t} = \mathrm{E}_t \left(\frac{K_{H,t}}{P_{H,t}^{-1} F_{H,t}} \right) \; ; \; \tilde{P}_{\mathcal{N},t} = \mathrm{E}_t \left(\frac{K_{\mathcal{N},t}}{P_{\mathcal{N},t}^{-1} F_{\mathcal{N},t}} \right)$$
$$\tilde{P}_{F,t} = \mathrm{E}_t \left(\frac{K_{F,t}}{P_{F,t}^{-1} F_{F,t}} \right) \; ; \; \tilde{P}^*_{N,t} = \mathrm{E}_t \left(\frac{K^*_{\mathcal{N},t}}{P^*_{\mathcal{N},t} F^*_{\mathcal{N},t}} \right), \tag{B.1}$$

with:

$$K_{H,t} \equiv \zeta \sum_{k=0}^{\infty} (P_{t+k} C_{t+k})^{-1} \tilde{Y}_{H,t+k} MC^n_{H,t+k} \quad ; \quad F_{H,t} \equiv P_{H,t} \sum_{k=0}^{\infty} (P_{t+k} C_{t+k})^{-1} \tilde{Y}_{H,t+k}$$
$$K_{\mathcal{N},t} \equiv \zeta \sum_{k=0}^{\infty} (P_{t+k} C_{t+k})^{-1} \tilde{Y}_{\mathcal{N},t+k} MC^n_{\mathcal{N},t+k} \quad ; \quad F_{\mathcal{N},t} \equiv P_{\mathcal{N},t} \sum_{k=0}^{\infty} (P_{t+k} C_{t+k})^{-1} \tilde{Y}_{\mathcal{N},t+k}$$
$$K_{F,t} \equiv \zeta \sum_{k=0}^{\infty} \left(P^*_{t+k} C^*_{t+k}\right)^{-1} \tilde{Y}_{F,t+k} MC^{n*}_{F,t+k} \quad ; \quad F_{F,t} \equiv P_{F,t} \sum_{k=0}^{\infty} \left(P^*_{t+k} C^*_{t+k}\right)^{-1} \tilde{Y}_{F,t+k}$$
$$K^*_{\mathcal{N},t} \equiv \zeta \sum_{k=0}^{\infty} \left(P^*_{t+k} C^*_{t+k}\right)^{-1} \tilde{Y}^*_{\mathcal{N},t+k} MC^{n*}_{H,t+k} \quad ; \quad F^*_{\mathcal{N},t} \equiv \sum_{k=0}^{\infty} \left(P^*_{t+k} C^*_{t+k}\right)^{-1} \tilde{Y}^*_{\mathcal{N},t+k}. \tag{B.2}$$

Eq.(B.2) implies that:

$$K_H = \frac{\zeta Y_H MC^n_H}{(1-\alpha\delta)(PC)} \quad ; \quad F_H = \frac{P_H Y_H}{(1-\alpha\delta)(PC)}$$
$$K_{\mathcal{N}} = \frac{\zeta Y_{\mathcal{N}} MC^n_{\mathcal{N}}}{(1-\alpha\delta)(PC)} \quad ; \quad F_{\mathcal{N}} = \frac{P_{\mathcal{N}} Y_{\mathcal{N}}}{(1-\alpha\delta)(PC)}$$
$$K_F = \frac{\zeta Y_F MC^n_F}{(1-\alpha\delta)(P^* C^*)} \quad ; \quad F_F = \frac{P_F Y_F}{(1-\alpha\delta)(P^* C^*)}$$
$$K^*_{\mathcal{N}} = \frac{\zeta Y^*_{\mathcal{N}} MC^{n*}_{\mathcal{N}}}{(1-\alpha\delta)(P^* C^*)} \quad ; \quad F_{\mathcal{N}} = \frac{P^*_{\mathcal{N}} Y^*_{\mathcal{N}}}{(1-\alpha\delta)(P^* C^*)}.$$

These equalities and Eq.(B.1) imply that:

$$P_H = \zeta MC^n \; ; \; P_{\mathcal{N}} = \zeta MC^n \; ; \; P_F = \zeta MC^{n*} \; ; \; P^*_{\mathcal{N}} = \zeta MC^{n*}, \tag{B.3}$$

where we use the equalities as follows:

$$MC^n_H = MC^n_{\mathcal{N}} \equiv MC^n \; ; \; MC^n_F = MC^{n*}_{\mathcal{N}} \equiv MC^{n*},$$

which are implied by Eq.(A.17). These equalities imply that:

$$P_H = P_{\mathcal{N}},$$
$$P_F = P^*_{\mathcal{N}}.$$

Combining these equalities and the definition of $P_{P,t}$, $P^*_{P,t}$, $P_{G,t}$ and $P^*_{G,t}$, we have:

$$P_P = P_H = P_{\mathcal{N}} = P_G,$$
$$P^*_P = P_F = P^*_{\mathcal{N}} = P^*_G. \tag{B.4}$$

Following Gali and Monacelli[18], we assume that PPP (purchasing power parity) holds in the steady state, which means that:

$$\mathsf{Q} = 1. \tag{B.5}$$

Eqs.(B.4) and (B.5) imply the following:

$$P = P^* = P_{\mathcal{T}} = P_{\mathcal{N}} = P^*_{\mathcal{N}} = P_H = P_F = P_P = P^*_P = P_G = P^*_G. \tag{B.6}$$

Note that because $P_F = P_H$ and $P_{\mathcal{N}} = P^*_{\mathcal{N}}$, we have:

$$\mathsf{T} = \mathsf{N} = 1. \tag{B.7}$$

Because of Eqs.(B.3), Eq.(B.4) can be rewritten as:

$$MC^n = MC^{n*}.$$

Thus, we have:

$$MC = MC^* = \zeta^{-1},$$

with $MC \equiv \frac{MC^n}{P}$ and $MC^* \equiv \frac{MC^{n*}}{P}$.

Furthermore, Eqs.(A.17) and (B.4) imply the following:

$$CN^{\varphi} = C^* \left(N^*\right)^{\varphi} = \frac{1-\tau}{\zeta}. \tag{B.8}$$

Eq.(B.8) implies the familiar expression:

$$\begin{aligned} (1-\tau)\, U_C\left(C\right) &= \zeta U_N\left(N\right), \\ (1-\tau)\, U_C\left(C^*\right) &= \zeta U_N\left(N^*\right). \end{aligned} \tag{B.9}$$

Note that because $\tau \in (0,1)$ and $\theta > 1$, this steady state is distorted.

Eq.(A.26) can be rewritten as:

$$Y_H = \gamma C + G_H \quad ; \quad Y_F = \gamma C + G_F, \tag{B.10}$$

by using Eq.(B.6). Because $G_H = G_F$, $Y_H = Y_F$. As with Eq.(B.6), Eq.(A.29) can be rewritten as:

$$Y_{\mathcal{N}} = (1-\gamma)\, C + G_{\mathcal{N}} \quad ; \quad Y^*_{\mathcal{N}} = (1-\gamma)\, C + G^*_{\mathcal{N}}. \tag{B.11}$$

Because $G_{\mathcal{N}} = G^*_{\mathcal{N}}$, $Y_{\mathcal{N}} = Y^*_{\mathcal{N}}$.

Eq.(B.6) and Eq.(18) in the text imply the following:

$$\begin{aligned} Y &= Y_H + Y_{\mathcal{N}} \quad ; \quad Y^* = Y_F + Y^*_{\mathcal{N}}, \\ G &= G_H + G_{\mathcal{N}} \quad ; \quad G^* = G_F + G^*_{\mathcal{N}}. \end{aligned} \tag{B.12}$$

Combining Eqs.(B.11) and (B.12), we have:

$$Y = C + G \quad ; \quad Y^* = C + G^*. \tag{B.13}$$

Because $G_H = G_F$ and $G_{\mathcal{N}} = G^*_{\mathcal{N}}$, Eq.(B.12) implies $G = G^*$. Thus,

$$Y = Y^*.$$

Eqs.(A.10) and (B.5) imply that:

$$C = C^*. \tag{B.14}$$

Eqs.(B.8) and (B.14) imply the following:

$$N = N^*.$$

Eq.(A.21) yields the following:

$$B\left(\frac{1-\delta}{\delta}\right) = \tau Y - G, \tag{B.15}$$

with $B \equiv \frac{B^n}{P}$. This equality implies $B = B^*$.

We assume $B > 0$; thus, another transversality condition for local government is given by:

$$\lim_{k\to\infty} \mathrm{E}_t \left[\delta^{k-t} U_C(C) RB\right] = 0, \tag{B.16}$$

which appears in footnote 9 in the text.

C Log-linearization of the Model

C.1 Aggregate Demand and Output

Log-linearizing Eq.(7) in the text, we obtain the following:

$$c^R_t = \mathsf{q}_t, \tag{C.1}$$

where q_t denotes the logarithmic CPI differential between the two countries.

Log-linearizing Eq.(A.7) and rearranging yields:

$$\mathsf{q}_t = (1-\gamma)\,\mathsf{n}_t. \tag{C.2}$$

Log-linearizing and manipulating Eq.(A.7), we obtain:

$$\pi_t = \gamma \pi_{\mathcal{T},t} + (1-\gamma)\,\pi_{\mathcal{N},t}, \tag{C.3}$$

with $\pi_{\mathcal{T},t} = \frac{1}{2}\pi_{H,t} + \frac{1}{2}\pi_{F,t}$ which is derived by log-linearizing the definition of the price index of tradables, where π_t denotes the CPI inflation rate in country H, $\pi_{\mathcal{T},t}$ denotes the tradable goods price inflation rate, $\pi_{H,t}$ and $\pi_{F,t}$ denote the inflation rates of tradables produced in countries H and F, respectively, and $\pi_{\mathcal{N},t}$ denotes the inflation rate of nontradables produced in country H.

Log-linearizing the definition of PPI, we have:

$$p_{P,t} = \gamma p_{H,t} + (1-\gamma)\, p_{\mathcal{N},t}. \tag{C.4}$$

This equality implies that:

$$\pi_{P,t} = \gamma \pi_{H,t} + (1-\gamma)\, \pi_{\mathcal{N},t}, \tag{C.5}$$

where $\pi_{P,t}$ denotes the PPI inflation rate in country H.

Log-linearizing Eq.(A.28), we have:

$$\begin{aligned} y_t &= \gamma y_{H,t} + \gamma p_{H,t} - \gamma p_{P,t} + (1-\gamma)\, y_{\mathcal{N},t} + (1-\gamma)\, p_{\mathcal{N},t} - (1-\gamma)\, p_{P,t} \\ &= \gamma y_{H,t} + (1-\gamma)\, y_{\mathcal{N},t} + \gamma p_{H,t} + (1-\gamma)\, p_{\mathcal{N},t} - p_{P,t}. \end{aligned}$$

Substituting Eq.(C.4) into this equality, we have:

$$\begin{aligned} y_t &= \gamma y_{H,t} + (1-\gamma)\, y_{\mathcal{N},t} + p_{P,t} - p_{P,t}, \\ &= \gamma y_{H,t} + (1-\gamma)\, y_{\mathcal{N},t}. \end{aligned} \tag{C.6}$$

Log-linearizing the definition of the average price of goods purchased by the government in country H yields:

$$p_{G,t} = \gamma p_{H,t} + (1-\gamma)\, p_{\mathcal{N},t}, \tag{C.7}$$

which implies that $p_{P,t} = p_{G,t}$.

Combining the log-linearized LHS of Eq.(A.29) and Eq.(C.7), we have:

$$g_t = \gamma g_{H,t} + (1-\gamma)\, g_{\mathcal{N},t}. \tag{C.8}$$

Log-linearizing the first equalities of Eqs.(A.26) and (A.27) and substituting these equalities into Eq.(C.6), we have:

$$y_t = (1-\sigma_G)\, c_t + \frac{(1-\sigma_G)\,\gamma}{2}\mathsf{t}_t + \frac{(1-\sigma_G)\,\psi}{2}\mathsf{n}_t + \sigma_G g_t. \tag{C.9}$$

Subtracting the counterpart of Eq.(C.9) in country F from Eq.(C.9), we have:

$$y_t^R = \gamma\,(1-\sigma_G)\,\mathsf{t}_t + (1-\gamma)\,\varpi\,(1-\sigma_G)\,\mathsf{n}_t + \sigma_G g_t^R. \tag{C.10}$$

Log-linearizing Eq.(A.23), we have:

$$\begin{aligned} b_t &= \mathrm{E}_t c_{t+1} - c_t - \frac{1}{\delta}\pi_t + \mathrm{E}_t \pi_{t+1} + \frac{1}{\delta}\hat{r}_{t-1} - \hat{r}_t + \frac{1}{\delta}b_{t-1} + \left(\frac{1-\delta}{\delta}\right)\frac{\gamma}{2}\mathsf{t}_t \\ &\quad - \frac{\tau}{\sigma_B}y_t + \frac{\sigma_G}{\sigma_B}g_t. \end{aligned} \tag{C.11}$$

Combining Eqs.(C.3), (C.6), (C.8), (C.10), (C.11) and the counterpart of Eq.(C.11), we have:

$$\begin{aligned} y_t^W &= \frac{\beta_W}{1-\sigma_G}\mathrm{E}_t y_{t+1}^W + \beta_W \mathrm{E}_t \pi_{t+1}^W - \beta_W \hat{r}_t + \frac{\beta_W}{\delta}\hat{r}_{t-1} - \beta_W b_t^W \\ &\quad + \frac{\beta_W}{\delta} b_{t-1}^W - \frac{\beta_W}{\delta}\pi_t^W + \sigma_G \nu_W g_t^W, \qquad \text{(C.12)} \\ y_t^R &= -\beta_R \delta b_t^R + \beta_R (1-\gamma) \upsilon \mathsf{n}_t - \beta_R (1-\gamma) \mathsf{n}_{t-1} + \beta_R b_{t-1}^R \\ &\quad + \sigma_G \nu_R g_t^R. \qquad \text{(C.13)} \end{aligned}$$

Log-linearizing Eq.(A.30) and substituting Eq.(C.9) yields:

$$\widehat{nx}_t = \frac{(1-\sigma_G)\,\psi}{2}\mathsf{n}_t,$$

with $\widehat{nx}_t \equiv \frac{dNX_t}{Y}$ denoting the percentage deviation of the net exports in country H from the steady-state value of output. Note that this equality becomes $\widehat{nx}_t = 0$ which implies that balanced trade is definitely applied, under our benchmark parameterization, $\eta = 1$.

C.2 Aggregate Supply and Inflation

Log-linearizing Eq.(A.16), we have:

$$\mathrm{E}_t\left[\sum_{k=0}^{\infty}(\alpha\delta)^k\left(\tilde{\mathsf{x}}_{H,t+k} + \mathsf{x}_{H,t+k} + \mathsf{x}_{\mathcal{T},t+k} - \mathsf{x}_{P,t+k} - mc_{H,t+k}\right)\right] = 0,$$

with $\tilde{\mathsf{x}}_{H,t+k} \equiv \ln \tilde{\mathsf{X}}_{H,t+k}$, $\mathsf{x}_{H,t+k} \equiv \ln \mathsf{X}_{H,t+k}$, $\mathsf{x}_{\mathcal{T},t+k} \equiv \ln \mathsf{X}_{\mathcal{T},t+k}$ and $\mathsf{x}_{P,t+k} \equiv \ln \mathsf{X}_{P,t+k}$.

Using the fact that $\tilde{\mathsf{x}}_{H,t+k} = \mathsf{x}_{H,t} - \sum_{s=1}^{k}\pi_{H,t+s}$, this can be rewritten as:

$$\mathrm{E}_t\left[\sum_{k=0}^{\infty}(\alpha\delta)^k\left(\tilde{\mathsf{x}}_{H,t} - \sum_{s=1}^{k}\pi_{H,t+s} + \mathsf{x}_{H,t+k} + \mathsf{x}_{\mathcal{T},t+k} - \mathsf{x}_{P+k,t} - mc_{H,t+k}\right)\right] = 0.$$

Furthermore, using the fact that $\sum_{k=0}^{\infty}(\alpha\delta)^k\sum_{s=1}^{k}\pi_{H,t+s} = \frac{1}{1-\alpha\delta}\sum_{k=1}^{\infty}(\alpha\delta)^k\pi_{H,t+k}$, this can be rewritten as:

$$\begin{aligned} &\frac{1}{1-\alpha\delta}\tilde{\mathsf{x}}_{H,t} - \frac{1}{1-\alpha\delta}\mathrm{E}_t\sum_{k=1}^{\infty}(\alpha\delta)^k\pi_{H,t+k} + \mathrm{E}_t\sum_{k=0}^{\infty}(\alpha\delta)^k\mathsf{x}_{H,t+k} + \mathrm{E}_t\sum_{k=0}^{\infty}(\alpha\delta)^k\mathsf{x}_{\mathcal{T},t+k}, \\ &\qquad -\mathrm{E}_t\sum_{k=0}^{\infty}(\alpha\delta)^k\mathsf{x}_{P,t+k} - \mathrm{E}_t\sum_{k=0}^{\infty}(\alpha\delta)^k mc_{H,t+k} = 0. \end{aligned}$$

Rearranging this, we have:

$$\tilde{\mathsf{x}}_{H,t} = \sum_{k=1}^{\infty}(\alpha\delta)^k\pi_{H,t+k} - (1-\alpha\delta)\sum_{k=0}^{\infty}(\alpha\delta)^k\mathsf{x}_{H,t+k} - (1-\alpha\delta)\sum_{k=0}^{\infty}(\alpha\delta)^k\mathsf{x}_{\mathcal{T},t+k},$$

$$+ (1-\alpha\delta) \sum_{k=0}^{\infty} (\alpha\delta)^k \mathsf{x}_{P+k,t} + (1-\alpha\delta) \sum_{k=0}^{\infty} (\alpha\delta)^k mc_{H,t+k},$$

$$= \alpha\delta\pi_{H,t+1} - (1-\alpha\delta)\mathsf{x}_{H,t} - (1-\alpha\delta)\mathsf{x}_{T,t} + (1-\alpha\delta)\mathsf{x}_{P,t},$$

$$+ (1-\alpha\delta) mc_{H,t} + \alpha\delta\tilde{\mathsf{x}}_{H,t+1}. \tag{C.14}$$

Log-linearizing the first equality of Eq.(A.13), we have:

$$\tilde{\mathsf{x}}_{H,t} = \frac{\alpha}{1-\alpha}\pi_{H,t}. \tag{C.15}$$

Combining Eqs.(C.14) and (C.15) yields:

$$\begin{aligned} \pi_{H,t} &= \delta\pi_{H,t+1} - \kappa\mathsf{x}_{H,t} - \kappa\mathsf{x}_{T,t} + \kappa\mathsf{x}_{P,t} + \kappa mc_{H,t}, \\ &= \delta\pi_{H,t+1} + (1-\gamma)\kappa p_{\mathcal{N},t} - (1-\gamma)\kappa p_{H,t} + \kappa mc_{H,t}. \end{aligned}$$

Taking the conditional expectation at t, the second equality can be rewritten as:

$$\pi_{H,t} = \delta \mathrm{E}_t \pi_{H,t+1} + \kappa(1-\gamma) p_{\mathcal{N},t} - \kappa(1-\gamma) p_{H,t} + \kappa mc_{H,t}. \tag{C.16}$$

Similar to Eq.(C.16), the log-linearized second equality of Eq.(A.14) is given by:

$$\pi_{\mathcal{N},t} = \delta \mathrm{E}_t \pi_{N,t+1} - \kappa\gamma p_{\mathcal{N},t} + \kappa\gamma p_{H,t} + \kappa mc_{\mathcal{N},t}. \tag{C.17}$$

Other FONCs for firms can be log-linearized similarly.

Substituting Eqs.(C.17) and (C.16) into Eq.(C.5), we have a PPI-based inflation dynamics equation as follows:

$$\pi_{P,t} = \delta \mathrm{E}_t \pi_{P,t+1} + \kappa mc_t, \tag{C.18}$$

where we use $mc_t = \gamma mc_{H,t} + (1-\gamma) mc_{\mathcal{N},t}$ which is derived by log-linearizing the definition of country-wide marginal cost.

Combining Eq.(C.17) and its counterpart for country F, the nontradables inflation differential is given by:

$$\pi^R_{\mathcal{N},t} = \delta \mathrm{E}_t \pi^R_{N,t+1} + \kappa\gamma \mathsf{n}_t - \kappa\gamma \mathsf{t}_t + \kappa mc^R_{\mathcal{N},t}, \tag{C.19}$$

with

$$\pi^R_{\mathcal{N},t} \equiv -(\mathsf{n}_t - \mathsf{n}_{t-1}), \tag{C.20}$$

being relative nontradables inflation.

By log-linearizing the first equalities in Eq.(A.12) and combining it with Eq.(C.6), we have:

$$y_t = \gamma a_{H,t} + (1-\gamma) a_{\mathcal{N},t} + n_t, \tag{C.21}$$

where we also use the log-linearized definition of hours of work, $n_t = \gamma n_{H,t} + (1-\gamma) n_{\mathcal{N},t}$.

Combining log-linearized Eq.(A.17), and Eqs.(C.9) and (C.21), we have:

$$\begin{aligned} mc_{H,t} &= \frac{\lambda}{1-\sigma_G} y_t - \frac{\psi}{2} \mathsf{n}_t - (1+\varphi\gamma) a_{H,t} - (1-\gamma)\varphi a_{\mathcal{N},t} - \frac{\sigma_G}{1-\sigma_G} g_t, \\ mc_{\mathcal{N},t} &= \frac{\lambda}{1-\sigma_G} y_t - \frac{\psi}{2} \mathsf{n}_t - \varphi\gamma a_{H,t} - [1+(1-\gamma)\varphi] a_{\mathcal{N},t} - \frac{\sigma_G}{1-\sigma_G} g_t. \end{aligned} \tag{C.22}$$

Substituting Eq.(C.22) into the log-linearized definition of the marginal cost $mc_t = \gamma mc_{H,t} + (1-\gamma) mc_{\mathcal{N},t}$, we have:

$$mc_t = \frac{\lambda}{1-\sigma_G} y_t - \frac{\psi}{2} \mathsf{n}_t - (1+\varphi)\gamma a_{H,t} - (1+\varphi)(1-\gamma) a_{\mathcal{N},t} - \frac{\sigma_G}{1-\sigma_G} g_t. \tag{C.23}$$

Combining the second equality in Eq.(C.22) and its counterpart for country F, the logarithmic marginal cost differential associated with nontradables is given by:

$$\begin{aligned} mc^R_{\mathcal{N},t} &= \frac{\lambda}{1-\sigma_G} y^R_t - \psi \mathsf{n}_t - \varphi\gamma a_{H,t} + \varphi\gamma a_{F,t} - [1+(1-\gamma)\varphi] a_{\mathcal{N},t}, \\ &\quad + [1+(1-\gamma)\varphi] a^*_{\mathcal{N},t} - \frac{\sigma_G}{1-\sigma_G} g^R_t. \end{aligned} \tag{C.24}$$

C.3 Marginal Cost and Output Gap

Following Gali[18], we define the relationship between output, its natural level and the output gap as:

$$y_t \equiv \bar{y}_t + \tilde{y}_t, \tag{C.25}$$

where $\tilde{y}_t$ denotes the logarithmic output gap measured from its natural level, and $\bar{y}_t$ denotes the logarithmic natural output level. Under the long-run equilibrium, $\tilde{y}_t = 0$ must hold.[31]

When the fiscal authorities design their policies to reduce the distortion generated by monopolistically competitive markets, real marginal costs under the long-run equilibrium are constant, and their logarithm is given by $mc_t = 0$. In addition, under the long-run equilibrium, PPP is applied.[32] Thus, the logarithmic NPD under the long-run equilibrium is given by $\mathsf{n}_t = 0$.

Combining these facts, Eq.(C.23) implies that:

$$\bar{y}_t = \bar{\beta}\gamma a_{H,t} + \bar{\beta}(1-\gamma) a_{\mathcal{N},t} + \frac{\sigma_G}{\lambda} g_t. \tag{C.26}$$

Combining Eqs.(C.12), (C.13), (C.25) and (C.26) can be rewritten as:

$$\tilde{y}^W_t = \frac{\beta_W}{1-\sigma_G} \mathrm{E}_t \tilde{y}^W_{t+1} - \beta_W \hat{r}_t + \beta_W \mathrm{E}_t \pi^W_{t+1} + \frac{\beta_W}{\delta} \hat{r}_{t-1} - \beta_W b^W_t + \frac{\beta_W}{\delta} b^W_{t-1},$$

[31] Following Gali[18], nominal rigidities disappear in the long-run equilibrium.

[32] Following Gali[18], we assume a steady state where PPP is applied.

$$
\begin{aligned}
& -\frac{\beta_W}{\delta}\pi_t^W - \frac{\gamma\bar{\beta}\beta_{\mathcal{T}}}{2}a_{H,t} - \frac{(1-\gamma)\,\bar{\beta}\beta_{\mathcal{N}}}{2}a_{\mathcal{N},t} - \frac{\gamma\bar{\beta}\beta_{\mathcal{T}}}{2}a_{F,t}, \\
& -\frac{(1-\gamma)\,\bar{\beta}\beta_{\mathcal{N}}}{2}a_{\mathcal{N},t}^{*} + \sigma_G\varsigma_W g_t^W, \qquad \text{(C.27)} \\
\tilde{y}_t^R \;=\; & -\beta_R\delta b_t^R + \beta_R(1-\gamma)\,\upsilon\mathsf{n}_t - \beta_R(1-\gamma)\,\mathsf{n}_{t-1} + \beta_R b_{t-1}^R - \bar{\beta}\gamma a_{H,t}, \\
& +\bar{\beta}\gamma a_{F,t} - \bar{\beta}(1-\gamma)\,a_{\mathcal{N},t} + \bar{\beta}(1-\gamma)\,a_{\mathcal{N},t}^{*} + \varsigma_R\sigma_G g_t^R, \qquad \text{(C.28)}
\end{aligned}
$$

which are Eqs.(13) and (14) in the text, respectively.

Combining Eqs.(C.18), (C.23), (C.25) and (C.26) we have:

$$
\pi_{P,t} = \delta \mathrm{E}_t\pi_{P,t+1} + \frac{\kappa\lambda}{1-\sigma_G}\tilde{y}_t - \frac{\psi\kappa}{2}\mathsf{n}_t, \qquad \text{(C.29)}
$$

which is Eq.(15) in the text. Similar to Eq.(C.29), we have the counterpart of Eq.(C.29) in country F.

C.4 NKRD

Combining Eqs.(C.19), (C.23), (C.25) and (C.26), we have:

$$
\begin{aligned}
\pi_{\mathcal{N},t}^R \;=\; & \delta \mathrm{E}_t\pi_{N,t+1}^R + \kappa\varphi\tilde{y}_t^R + \kappa\mathsf{n}_t - \kappa\varphi\gamma\left(1-\bar{\beta}\right)a_{H,t} + \kappa\varphi\gamma\left(1-\bar{\beta}\right)a_{F,t}, \\
& -\kappa\left[1+\varphi(1-\gamma)\left(1-\bar{\beta}\right)\right]a_{\mathcal{N},t} + \kappa\left[1+\varphi(1-\gamma)\left(1-\bar{\beta}\right)\right]a_{\mathcal{N},t}^{*}, \\
& -\frac{\kappa\sigma_G}{1-\sigma_G}\left(1-\frac{\varphi}{\lambda}\right)g_t^R, \qquad \text{(C.30)}
\end{aligned}
$$

which is Eq.(16) in the text.

D Welfare Criterion

Following Gali and Monacelli[18], Gali[17] and Benigno and Woodford[10], we show the derivation of the welfare criterion in the text based on the second-order approximated utility function of Eq.(A.1) in the present appendix. $\eta = 1$ is assumed through the present appendix.

This section consists of four subsections. Subsection D.1 presents the second-order Taylor expansion of the utility function. Subsection D.2 presents the second-order approximation of the FONCs for firms. Subsection D.3 eliminates the linear term and completes the derivation of the welfare criterion. Subsection D.4 discusses other details regarding the coefficients and the NKPC in terms of the welfare-relevant output gap.

D.1 Step 1: The Second-order Taylor Expansion of the Utility Function

The second-order Taylor expansion of the period utility function in Eq.(1) in the text is given by:

$$
\frac{U_t - U}{U_C C} = C^{-1}\left[C\left(c_t + \frac{1}{2}c_t^2\right) + \frac{1}{2}\frac{U_{CC}}{U_C}C^2c_t^2 - \frac{U_N}{U_C}N\left(n_t + \frac{1}{2}n_t^2\right) + \frac{1}{2}\frac{U_{NN}}{U_C}N^2n_t^2\right]
$$

$$
\begin{aligned}
& + o\left(\|\xi\|^3\right) \\
= \quad & c_t + \frac{1}{2}c_t^2 + \frac{1}{2}\frac{U_{CC}}{U_C}Cc_t^2 - \frac{U_N}{U_C}\frac{N}{C}\left(n_t + \frac{1}{2}n_t^2\right) + \frac{U_{NN}}{U_C}\frac{N^2}{C}n_t^2 \\
& + o\left(\|\xi\|^3\right) \qquad \text{(D.1)}
\end{aligned}
$$

where we assume that utility is separable by consumption and hours of work, i.e., $U_{CN} = 0$. Plugging $U_C = C^{-1}$, $U_{CC} = -C^{-2}$, $U_N = N^{\varphi}$ and $U_{NN} = \varphi N^{\varphi - 1}$ into Eq.(D.1), we have:

$$
\begin{aligned}
\frac{U_t - U}{U_C C} &= c_t - \frac{U_N N}{U_C C}\left(n_t + \frac{1+\varphi}{2}n_t^2\right) + o\left(\|\xi\|^3\right) \\
&= \Phi\frac{N}{C}n_t + c_t - \frac{N}{C}\left[n_t + \frac{1+\varphi}{2}(1+\Phi)\,n_t^2\right] + o\left(\|\xi\|^3\right), \qquad \text{(D.2)}
\end{aligned}
$$

where we use the fact that $1 - \Phi = \frac{1-\tau}{\zeta}$ and Eq.(B.9).

Likewise, we have:

$$
\frac{U_t^* - U}{U_C C} = \Phi\frac{N}{C}n_t^* + c_t^* - \frac{N}{C}\left[n_t^* + \frac{1+\varphi}{2}(1+\Phi)\,(n_t^*)^2\right] + o\left(\|\xi\|^3\right). \qquad \text{(D.3)}
$$

Eq.(A.12) can be rewritten as:

$$
N_{H,t} = \frac{Y_{H,t}\mathsf{D}_{H,t}}{A_{\mathcal{N},t}} \quad N_{\mathcal{N},t} = \frac{Y_{\mathcal{N},t}\mathsf{D}_{\mathcal{N},t}}{A_{\mathcal{N},t}},
$$

with $\mathsf{D}_{H,t} \equiv \int_0^1 \left(\frac{P_{H,t}(h)}{P_{H,t}}\right)^{-\theta} dh$ and $\mathsf{D}_{\mathcal{N},t} \equiv \int_0^1 \left(\frac{P_{\mathcal{N},t}(h)}{P_{\mathcal{N},t}}\right)^{-\theta} dh$ where we use the fact that $\frac{\int_0^1 Y_{H,t}(h)dh}{Y_{H,t}} = \mathsf{D}_{H,t}$ and $\frac{\int_0^1 Y_{\mathcal{N},t}(h)dh}{Y_{\mathcal{N},t}} = \mathsf{D}_{\mathcal{N},t}$.

Log-linearizing these equalities, we obtain:

$$
n_{H,t} = y_{H,t} + \mathsf{d}_{H,t} + \text{t.i.p}, \quad ; \quad n_{\mathcal{N},t} = y_{\mathcal{N},t} + \mathsf{d}_{\mathcal{N},t} + \text{t.i.p.}
$$

Combining these equalities with Eq.(C.6) and the log-linearized definition of country level hours of work, $n_t = \gamma n_{H,t} + (1-\gamma)\,n_{\mathcal{N},t}$ yields:

$$
n_t = y_t + \gamma \mathsf{d}_{H,t} + (1-\gamma)\,\mathsf{d}_{\mathsf{N},t} + \text{t.i.p.} \qquad \text{(D.4)}
$$

Let $P_{P,t}(h) \equiv \frac{P_{H,t}(h)Y_{H,t}(h) + P_{\mathcal{N},t}(h)Y_{\mathcal{N},t}(h)}{Y_{H,t}(h) + Y_{\mathcal{N},t}(h)}$ and $P_{P,t}^*(f) \equiv \frac{P_{F,t}(f)Y_{F,t}(f) + P_{\mathcal{N},t}^*(f)Y_{\mathcal{N},t}^*(f)}{Y_{F,t}(h) + Y_{\mathcal{N},t}^*(h)}$. These yield $p_{P,t}(h) = \gamma p_{H,t} + (1-\gamma)\,p_{\mathcal{N},t}(h)$ and $p_{P,t}^*(f) = \gamma p_{f,t} + (1-\gamma)\,p_{\mathcal{N},t}^*(f)$ by log-linearizing. Taking these equalities, Eq.(D.4) can be rewritten as:

$$
\begin{aligned}
n_t &= y_t + \gamma \ln \mathrm{E}_h\left(\frac{P_{H,t}(h)}{P_{H,t}}\right)^{-\theta} + (1-\gamma)\ln \mathrm{E}_h\left(\frac{P_{\mathcal{N},t}(h)}{P_{\mathcal{N},t}}\right)^{-\theta} + \text{t.i.p}, \\
&= y_t - \theta \mathrm{E}_h\left[\gamma \ln\left(\frac{P_{H,t}(h)}{P_{H,t}}\right) + \ln(1-\gamma)\left(\frac{P_{\mathcal{N},t}(h)}{P_{\mathcal{N},t}}\right)\right] + \text{t.i.p},
\end{aligned}
$$

$$
\begin{aligned}
&= y_t - \theta \mathrm{E}_h \left[\gamma \left(p_{H,t}(h) - p_{H,t}\right) + (1-\gamma)\left(p_{\mathcal{N},t}(h) - p_{\mathcal{N},t}\right)\right] + a_t, \\
&= y_t - \theta \ln \mathrm{E}_h \left(\frac{P_{P,t}(h)}{P_{P,t}}\right) + \text{t.i.p}, \\
&= y_t + \ln \int_0^1 \left(\frac{P_{P,t}(h)}{P_{P,t}}\right)^{-\theta} dh + \text{t.i.p}, \\
&= y_t + \mathsf{d}_t + \text{t.i.p}, \qquad \text{(D.5)}
\end{aligned}
$$

with $\mathsf{D}_t \equiv \int_0^1 \left(\frac{P_{P,t}(h)}{P_{P,t}}\right)^{-\theta} dh$. Likewise, we have:

$$
n_t^* = y_t^* + \mathsf{d}_t^* + \text{t.i.p.} \qquad \text{(D.6)}
$$

Substituting Eqs.(D.5) and (D.6) into Eqs.(D.2) and (D.3), we have:

$$
\begin{aligned}
\frac{U_t - U}{U_C C} &= \frac{\Phi}{1-\sigma_G} y_t + c_t - \frac{1}{1-\sigma_G}\left[y_t + (1+\Phi)\,\mathsf{d}_t + \frac{(1+\varphi)(1+\Phi)}{2}\left(y_t^2 - 2y_t a_t\right)\right], \\
&\quad + \text{t.i.p.} + o\left(\|\xi\|^3\right), \\
\frac{U_t^* - U}{U_C C} &= \frac{\Phi}{1-\sigma_G} y_t^* + c_t^* - \frac{1}{1-\sigma_G}\left\{y_t^* + (1+\Phi)\,\mathsf{d}_t^* + \frac{(1+\varphi)(1+\Phi)}{2}\left[\left(y_t^*\right)^2 - 2y_t^* a_t^*\right]\right\}, \\
&\quad + \text{t.i.p.} + o\left(\|\xi\|^3\right), \qquad \text{(D.7)}
\end{aligned}
$$

with $a_t \equiv \gamma a_{H,t} + (1-\gamma)\, a_{\mathcal{N},t}$ and $a_t^* \equiv \gamma a_{F,t} + (1-\gamma)\, a_{\mathcal{N},t}^*$ where we use the fact that $\frac{N}{C} = (1-\sigma_G)^{-1}$ because $N = Y$.

Combining Eqs.(C.1), (C.2), (C.9) and (C.10), we have:

$$
c_t = \frac{1}{1-\sigma_G} y_t + \frac{1}{1-\sigma_G} y_t^* - c_t^* + \frac{2\sigma_G}{1-\sigma_G} g_t^W.
$$

Combining Eq.(D.7) and this equality yields:

$$
\begin{aligned}
\frac{U_t^W - U}{U_C C} &= \frac{\Phi}{1-\sigma_G} y_t^W - \frac{1}{(1-\sigma_G)\,2}\left\{(1+\Phi)\left(\mathsf{d}_t + \mathsf{d}_t^*\right) + \frac{(1+\varphi)(1+\Phi)}{2}\left[y_t^2\right.\right., \\
&\quad \left.\left. -2y_t a_t + \left(y_t^*\right)^2 - 2y_t^* a_t^*\right]\right\} + \text{t.i.p.} + o\left(\|\xi\|^3\right). \qquad \text{(D.8)}
\end{aligned}
$$

Let $\hat{p}_{P,t}(h) \equiv p_{P,t}(h) - p_{P,t}$. As derived by Gali and Monacelli[18], note that:

$$
\begin{aligned}
\left(\frac{P_{P,t}(h)}{P_{P,t}}\right)^{1-\theta} &= \exp\left[(1-\theta)\,\hat{p}_{P,t}(h)\right], \\
&= 1 - (1-\theta)\,\hat{p}_{P,t}(h) + \frac{(1-\theta)^2}{2}\hat{p}_{P,t}(h) + o\left(\|\xi\|^3\right) \qquad \text{(D.9)}
\end{aligned}
$$

In the symmetric equilibrium, we have $\frac{P_{P,t}(h)}{P_{P,t}} = 1$. This implies:

$$
\mathrm{E}_h \left(\frac{P_{P,t}(h)}{P_{P,t}}\right)^{1-\theta} = 1. \qquad \text{(D.10)}
$$

Combining Eqs.(D.9) and (D.10), we have:

$$\mathrm{E}_h \hat{p}_{P,t}(h) = \frac{\theta - 1}{2} \mathrm{E}_h \hat{p}_{P,t}(h)^2 . \tag{D.11}$$

In addition, the second-order approximation to $\left(\frac{P_{P,t}(h)}{P_{P,t}}\right)^{-\theta}$ yields:

$$\left(\frac{P_{P,t}(h)}{P_{P,t}}\right)^{-\theta} = 1 - \theta \hat{p}_{P,t}(h) + \frac{\theta^2}{2} \hat{p}_{P,t}(h)^2 + o\left(\|\xi\|^3\right).$$

This equality implies:

$$\mathsf{D}_t = 1 - \theta \mathrm{E}_h \hat{p}_{P,t}(h) + \frac{\theta^2}{2} \mathrm{E}_h \hat{p}_{P,t}(h)^2 + o\left(\|\xi\|^3\right).$$

Substituting Eq.(D.11) into this equality, we have:

$$\begin{aligned} \mathsf{D}_t &= 1 + \frac{\theta}{2} \mathrm{E}_h \hat{p}_{P,t}(h)^2 + o\left(\|\xi\|^3\right) \\ &= 1 + \frac{\theta}{2} \mathrm{var}_h\left(\hat{p}_{P,t}(h)\right) + o\left(\|\xi\|^3\right). \end{aligned}$$

This equality implies:

$$\mathsf{d}_t = \frac{\theta}{2} \mathrm{var}_h\left(p_{P,t}(h)\right) + o\left(\|\xi\|^3\right), \tag{D.12}$$

which clearly corresponds to the equality derived by Gali and Monacelli[18].

Lemma 1

$$\sum_{t=0}^{\infty} \delta^t \mathrm{var}_h\left(p_{P,t}(h)\right) = \frac{1}{\kappa} \sum_{t=0}^{\infty} \delta^t \pi_{P,t}^2$$

Proof: See Woodford[35], p 399–400.

Substituting *Lemma 1*, Eqs.(D.12) and (D.8) into the definition of welfare in the text, we have:

$$\begin{aligned} \mathcal{W}^W = \; & \mathrm{E}_0 \sum_{t=0}^{\infty} \delta^t \left[\frac{\Phi}{1 - \sigma_G} \tilde{y}_t^W - \frac{(1+\Phi)\theta}{(1-\sigma_G)4} \pi_{P,t}^2 - \frac{(1+\Phi)\theta}{(1-\sigma_G)4} \left(\pi_{P,t}^*\right)^2 \right. \\ & \left. - \frac{(1+\varphi)(1+\Phi)}{(1-\sigma_G)4} (y_t - a_t)^2 - \frac{(1+\varphi)(1+\Phi)}{(1-\sigma_G)4} (y_t^* - a_t^*)^2 \right] \\ & + \mathrm{t.i.p.} + o\left(\|\xi\|^3\right). \end{aligned} \tag{D.13}$$

Note that $\mathrm{E}_0 \sum_{k=0}^{\infty} \delta^t \frac{U_t^W - U}{U_C C} = \mathcal{W}^W$ because $U_C C = 1$ and $U = U^*$.

D.2 Step 2: The Second-order Approximation of the FONCs for Firms

Substituting the first and the second equalities in Eq.(B.1) into the first and the second equalities in Eq.(A.13), we have:

$$\begin{aligned}\frac{1}{1-\alpha}\left(1-\alpha\Pi_{H,t}^{\theta-1}\right) &= \left(\frac{F_{H,t}}{K_{H,t}}\right)^{\theta-1}\\ \frac{1}{1-\alpha}\left(1-\alpha\Pi_{\mathcal{N},t}^{\theta-1}\right) &= \left(\frac{F_{\mathcal{N},t}}{K_{\mathcal{N},t}}\right)^{\theta-1}\end{aligned} \qquad (D.14)$$

Taking logarithms on both sides in Eq.(D.14), we have:

$$\begin{aligned}-\log\left(\frac{1}{1-\alpha}-\frac{\alpha}{1-\alpha}\Pi_{H,t}^{\theta-1}\right) &= (\theta-1)\left(\log K_{H,t}-\log F_{H,t}\right)\\ -\log\left(\frac{1}{1-\alpha}-\frac{\alpha}{1-\alpha}\Pi_{\mathcal{N},t}^{\theta-1}\right) &= (\theta-1)\left(\log K_{\mathcal{N},t}-\log F_{\mathcal{N},t}\right)\end{aligned} \qquad (D.15)$$

The first-order approximation of the LHS in Eq.(D.15) is given by:

$$\begin{aligned}-\log\left(\frac{1}{1-\alpha}-\frac{\alpha}{1-\alpha}\Pi_{H,t}^{\theta-1}\right) &= \frac{(\theta-1)\,\alpha}{1-\alpha}\pi_{H,t}+o\left(\|\xi\|^2\right)\\ -\log\left(\frac{1}{1-\alpha}-\frac{\alpha}{1-\alpha}\Pi_{\mathcal{N},t}^{\theta-1}\right) &= \frac{(\theta-1)\,\alpha}{1-\alpha}\pi_{\mathcal{N},t}+o\left(\|\xi\|^2\right)\end{aligned} \qquad (D.16)$$

A weighted average of the two equalities in Eq.(D.16) is given by:

$$\frac{(\theta-1)\,\alpha}{1-\alpha}\gamma\pi_{H,t}+\frac{(\theta-1)\,\alpha}{1-\alpha}(1-\gamma)\,\pi_{\mathcal{N},t}=\frac{(\theta-1)\,\alpha}{1-\alpha}\pi_{P,t},$$

where we use Eq.(C.5). The second-order approximation of the RHS of this equality yields:

$$\frac{(\theta-1)\,\alpha}{1-\alpha}\pi_{P,t}=\frac{(\theta-1)\,\alpha}{1-\alpha}\pi_{P,t}+\frac{(\theta-1)\,\alpha 3}{(1-\alpha)\,4}\pi_{P,t}^2+o\left(\|\xi\|^3\right).$$

Combining this equality and Eq.(D.15), we have:

$$\frac{(\theta-1)\,\alpha}{1-\alpha}\pi_{P,t}+\frac{(\theta-1)\,\alpha 3}{(1-\alpha)\,4}\pi_{P,t}^2=(\theta-1)\,(k_t-f_t)+o\left(\|\xi\|^3\right), \qquad (D.17)$$

with $k_t \equiv \gamma k_{H,t}+(1-\gamma)\,k_{\mathcal{N},t}$ and $f_t \equiv \gamma f_{H,t}+(1-\gamma)\,f_{\mathcal{N},t}$ where we use the fact that $K=F$. Note that $K_{H,t}$, $K_{\mathcal{N},t}$, $F_{H,t}$ and $F_{\mathcal{N},t}$ are $\left(\|\xi\|^2\right)$.

Log-linearizing the first and the second equality in the LHS in Eq.(B.2) and combining them, we have:

$$k_t=\tilde{k}_t-\frac{\theta\alpha}{1-\alpha}\pi_{P,t}, \qquad (D.18)$$

with $\tilde{k}_t \equiv (1-\alpha\delta)\,\mathrm{E}_t \sum_{k=0}^{\infty} (\alpha\delta)^k \, \tilde{k}_{t,t+k}$ and $\tilde{k}_{t,t+k} \equiv (1+\varphi)\, y_{t+k} - \frac{(1+\varphi)\sigma_G}{\lambda} a_{t+k} + \frac{(1+\varphi)\sigma_G}{\lambda} g_{t+k} + \theta \sum_{s=1}^{k} \pi_{P,t+s}$.

Log-linearizing the first and the second equality on the RHS in Eq.(B.2) and combining them, we have:

$$f_t = \tilde{f}_t - \frac{\theta\alpha}{1-\alpha}\pi_{P,t}, \tag{D.19}$$

with $\tilde{f}_t \equiv (1-\alpha\delta)\,\mathrm{E}_t \sum_{k=0}^{\infty} (\alpha\delta)^k \, \tilde{f}_{t,t+k}$ and $\tilde{f}_{t,t+k} \equiv -\frac{\sigma_G}{1-\sigma_G} y_{t+k} - \frac{(1+\varphi)\sigma_G}{\lambda} a_{t+k} + \frac{(1+\varphi)\sigma_G}{\lambda} g_{t+k} + (\theta - 1) \sum_{s=1}^{k} \pi_{P,t+s}$.

Subtracting Eq.(D.19) from Eq.(D.18) yields:

$$k_t - f_t = \tilde{k}_t - \tilde{f}_t. \tag{D.20}$$

Substituting Eq.(D.20) into Eq.(D.17) yields:

$$\frac{(\theta-1)\,\alpha}{1-\alpha}\pi_{P,t} + \frac{(\theta-1)\,\alpha 3}{(1-\alpha)\,4}\pi_{P,t}^2 = (\theta-1)\left(\tilde{k}_t - \tilde{f}_t\right) + o\left(\|\xi\|^3\right). \tag{D.21}$$

An arbitrary variable V_t can be approximated as:

$$\begin{aligned} V_t &= e^{\ln V_t} \\ &= e^{\ln V} + e^{\ln V}\left(\ln V_t - \ln V\right) + \frac{1}{2} e^{\ln V}\left(\ln V_t - \ln V\right)^2 + o\left(\|\xi\|^3\right) \\ &= V\left(1 + v_t + \frac{1}{2}v_t^2\right) + o\left(\|\xi\|^3\right). \end{aligned}$$

Thus, we have the second-order approximation of $\tilde{k}_t$ and $\tilde{f}_t$ as follows:

$$\begin{aligned} \tilde{k}_t &= \tilde{k}_t + \frac{1}{2}\tilde{k}_t^2 \\ &= (1-\alpha\delta)\,\mathrm{E}_t \sum_{k=0}^{\infty} (\alpha\delta)^k \left(\tilde{k}_{t,t+k} + \frac{1}{2}\tilde{k}_{t,t+k}^2\right) && \text{(D.22)} \\ \tilde{f}_t &= \tilde{f}_t + \frac{1}{2}\tilde{f}_t^2 \\ &= (1-\alpha\delta)\,\mathrm{E}_t \sum_{k=0}^{\infty} (\alpha\delta)^k \left(\tilde{f}_{t,t+k} + \frac{1}{2}\tilde{f}_{t,t+k}^2\right) && \text{(D.23)} \end{aligned}$$

Subtracting Eq.(D.23) from Eq.(D.22) yields:

$$\begin{aligned} \tilde{k}_t - \tilde{f}_t &= (1-\alpha\delta)\,\mathrm{E}_t \sum_{k=0}^{\infty} (\alpha\delta)^k \left[\left(\tilde{k}_{t,t+k} - \tilde{f}_{t,t+k}\right) + \frac{1}{2}\left(\tilde{k}_{t,t+k}^2 - \tilde{f}_{t,t+k}^2\right)\right] \\ &\quad - \frac{(1-\alpha\delta)\,\alpha}{2\,(1-\alpha)}\pi_{P,t} Z_t + o\left(\|\xi\|^3\right) && \text{(D.24)} \end{aligned}$$

with

$$Z_t \equiv \mathrm{E}_t \sum_{k=0}^{\infty} (\alpha\delta)^k \left(\tilde{k}_{t,t+k} + \tilde{f}_{t,t+k}\right). \tag{D.25}$$

The first term on the RHS in Eq.(D.24) can be rewritten as:

$$\mathrm{E}_t \sum_{k=0}^{\infty} (\alpha\delta)^k \left(\tilde{k}_{t,t+k} - \tilde{f}_{t,t+k}\right) = \mathrm{E}_t \sum_{k=0}^{\infty} (\alpha\delta)^k \frac{\lambda}{1-\sigma_G} \tilde{y}_{t+k} + \frac{1}{1-\alpha\delta} \mathcal{P}_{P,t} \tag{D.26}$$

with $\mathcal{P}_{P,t} \equiv \mathrm{E}_t \sum_{k=0}^{\infty} (\alpha\delta)^k \pi_{P,t+k}$.

The second term on the RHS in Eq.(D.24) can be rewritten as:

$$\begin{aligned} \frac{1}{2}\mathrm{E}_t \sum_{k=0}^{\infty} (\alpha\delta)^k \left(\tilde{k}^2_{t,t+k} - \tilde{f}^2_{t,t+k}\right) &= \frac{1}{2}\mathrm{E}_t \sum_{k=0}^{\infty} (\alpha\delta)^k \left(\widetilde{kk}^2_{t,t+k} - \widetilde{ff}^2_{t,t+k}\right) \\ &+ \mathrm{E}_t \sum_{k=0}^{\infty} (\alpha\delta)^k \sum_{s=1}^{k} \pi_{P,t+k} \left[\theta \widetilde{kk}_{t,t+k} - (\theta-1) \widetilde{ff}_{t,t+k}\right] \\ &+ \frac{1}{2}\mathrm{E}_t \sum_{k=0}^{\infty} (\alpha\delta)^k \left(\sum_{s=1}^{k} \pi_{P,t+s}\right)^2 \left[\theta^2 - (\theta-1)^2\right] \end{aligned} \tag{D.27}$$

with

$$\begin{aligned} \widetilde{kk}_{t,t+k} &\equiv (1+\varphi)\,\tilde{y}_{t+k} - \frac{\sigma_G (1+\varphi)}{\lambda} a_{t+k} + \frac{\sigma_G (1+\varphi)}{\lambda} g_{t+k}, \\ \widetilde{ff}_{t,t+k} &\equiv -\frac{\sigma_G}{1-\sigma_G} \tilde{y}_{t+k} - \frac{\sigma_G (1+\varphi)}{\lambda} a_{t+k} + \frac{\sigma_G (1+\varphi)}{\lambda} g_{t+k}. \end{aligned} \tag{D.28}$$

The last term on the RHS in Eq.(D.27) can be rewritten as:

$$\frac{1}{2}\mathrm{E}_t \sum_{k=0}^{\infty} (\alpha\delta)^k \left(\sum_{s=1}^{k} \pi_{P,t+s}\right)^2 \left[\theta^2 - (\theta-1)^2\right] = \frac{2\theta-1}{(1-\alpha\delta)\,2} \mathrm{E}_t \sum_{k=0}^{\infty} (\alpha\delta)^k \pi_{P,t+k} \left(\pi_{P,t+k} + 2\mathcal{P}_{P,t+k}\right). \tag{D.29}$$

Furthermore, the second term on the RHS in Eq.(D.29) can be rewritten as:

$$\mathrm{E}_t \sum_{k=0}^{\infty} (\alpha\delta)^k \sum_{s=1}^{k} \pi_{P,t+s} \left[\theta \widetilde{kk}_{t,t+k} - (\theta-1) \widetilde{ff}_{t,t+k}\right] = \mathrm{E}_t \sum_{k=0}^{\infty} (\alpha\delta)^k \pi_{P,t+k} J_{t+k}, \tag{D.30}$$

with

$$J_t \equiv \mathrm{E}_t \sum_{k=0}^{\infty} (\alpha\delta)^k \left[\theta \widetilde{kk}_{t,t+k} - (\theta-1) \widetilde{ff}_{t,t+k}\right]. \tag{D.31}$$

Substituting Eqs.(D.29) and (D.30) into Eq.(D.28) yields:

$$\begin{aligned}\frac{1}{2}\mathrm{E}_t\sum_{k=0}^{\infty}(\alpha\delta)^k\left(\tilde{k}_{t,t+k}^2-\tilde{f}_{t,t+k}^2\right) &= \frac{1}{2}\mathrm{E}_t\sum_{k=0}^{\infty}(\alpha\delta)^k\left(\widetilde{kk}_{t,t+k}^2-\widetilde{ff}_{t,t+k}^2\right)\\ &\quad+\mathrm{E}_t\sum_{k=0}^{\infty}(\alpha\delta)^k\pi_{P,t+k}J_{t+k} \qquad\qquad (\mathrm{D.32})\\ &\quad+\frac{2\theta-1}{(1-\alpha\delta)\,2}\mathrm{E}_t\sum_{k=0}^{\infty}(\alpha\delta)^k\pi_{P,t+k}\left(\pi_{P,t+k}+2\mathcal{P}_{P,t+k}\right).\end{aligned}$$

Substituting Eqs.(D.25) and (D.31) into Eq.(D.24), we have:

$$\begin{aligned}\tilde{k}_t-\tilde{f}_t &= \mathrm{E}_t\sum_{k=0}^{\infty}(\alpha\delta)^k\left\{(1-\alpha\delta)\left[\left(\widetilde{kk}_{t,t+k}-\widetilde{ff}_{t,t+k}\right)+\frac{1}{2}\left(\widetilde{kk}_{t,t+k}^2-\widetilde{ff}_{t,t+k}^2\right)\right]\right\}\\ &\quad+\mathrm{E}_t\sum_{k=1}^{\infty}(\alpha\delta)^k\pi_{P,t+k}+(1-\alpha\delta)\,\mathrm{E}_t\sum_{k=0}^{\infty}(\alpha\delta)^k\pi_{P,t+k}J_{t+k}\\ &\quad+\frac{2\theta-1}{2}\mathrm{E}_t\sum_{k=0}^{\infty}(\alpha\delta)^k\left(\pi_{P,t+k}+2\mathcal{P}_{P,t+k}\right)-\frac{(1-\alpha\delta)\,\alpha}{2\,(1-\alpha)}\pi_{P,t}Z_t+o\left(\|\xi\|^3\right).\end{aligned} \qquad (\mathrm{D.33})$$

Substituting Eq.(D.21) into Eq.(D.33) to eliminate the term $\tilde{k}_t-\tilde{f}_t$ in the LHS in Eq.(D.32) yields:

$$\begin{aligned}\pi_{P,t}+\frac{3}{4}\pi_{P,t}^2+\frac{1-\alpha\delta}{2}\pi_{P,t}Z_t &= \kappa\left(\widetilde{kk}_{t,t+k}-\widetilde{ff}_{t,t+k}\right)+\frac{(1-\alpha)(1-\alpha\delta)}{2\alpha}\left(\widetilde{kk}_{t,t+k}^2-\widetilde{ff}_{t,t+k}^2\right)\\ &\quad+(1-\alpha)\,\delta\mathrm{E}_t\pi_{P,t+1}+(1-\alpha)(1-\alpha\delta)\,\mathrm{E}_t\pi_{P,t+1}J_{t+1}\\ &\quad+\frac{(1-\alpha)(2\theta-1)}{2}\delta\mathrm{E}_t\pi_{P,t+1}\left(\pi_{P,t+1}+2\mathcal{P}_{P,t+1}\right)\\ &\quad+\alpha\delta\mathrm{E}_t\left(\pi_{P,t+1}+\frac{3}{4}\pi_{P,t+1}^2+\frac{1-\alpha\delta}{2}\pi_{P,t+1}Z_{t+1}\right)\\ &\quad+o\left(\|\xi\|^3\right).\end{aligned} \qquad (\mathrm{D.34})$$

Eq.(D.31) can be rewritten as:

$$J_t=\frac{1}{2}\mathrm{E}_t\sum_{k=0}^{\infty}(\alpha\delta)^k\left[\widetilde{kk}_{t,t+k}-\widetilde{ff}_{t,t+k}+(2\theta-1)\left(\widetilde{kk}_{t,t+k}-\widetilde{ff}_{t,t+k}\right)\right]. \qquad (\mathrm{D.35})$$

Substituting Eqs.(D.18), (D.19) and (D.28) into Eq.(D.25), we have:

$$\mathrm{E}_t\sum_{k=0}^{\infty}(\alpha\delta)^k\left(\widetilde{kk}_{t,t+k}-\widetilde{ff}_{t,t+k}\right)=Z_t-\frac{2\theta-1}{1-\alpha\delta}\mathcal{P}_{P,t}. \qquad (\mathrm{D.36})$$

Substituting Eq.(D.28) into Eq.(D.26) yields:

$$\mathrm{E}_t \sum_{k=0}^{\infty} (\alpha\delta)^k \left(\widetilde{kk}_{t,t+k} - \widetilde{ff}_{t,t+k}\right) = \mathrm{E}_t \sum_{k=0}^{\infty} (\alpha\delta)^k \left(\tilde{k}_{t,t+k} - \tilde{f}_{t,t+k}\right) - \frac{1}{1-\alpha\delta}\mathcal{P}_{P,t}, \tag{D.37}$$

where we use the fact that

$$\widetilde{kk}_{t,t} - \widetilde{ff}_{t,t} = \frac{\lambda}{1-\sigma_G}\tilde{y}_t. \tag{D.38}$$

Substituting Eqs.(D.36) and (D.37) into Eq.(D.35) yields:

$$J_t = \frac{1}{2}\mathrm{E}_t \sum_{k=0}^{\infty} (\alpha\delta)^k \left[Z_t + (2\theta - 1)\left(\tilde{k}_{t,t+k} - \tilde{f}_{t,t+k}\right) - \frac{2(\theta-1)}{1-\alpha\delta}\mathcal{P}_{P,t}\right]. \tag{D.39}$$

In the first order, Eq.(D.21) can be rewritten as:

$$\pi_{P,t} = \kappa \mathrm{E}_t \sum_{k=0}^{\infty} (\alpha\delta)^k \left(\tilde{k}_{t,t+k} - \tilde{f}_{t,t+k}\right) + o\left(\|\xi\|^2\right).$$

Substituting this equality into Eq.(D.39), we have:

$$J_t = \frac{1}{2}Z_t + \frac{\alpha(2\theta-1)}{2\kappa}\pi_{P,t} - \frac{2\theta-1}{1-\alpha\delta}\mathcal{P}_{P,t}.$$

Substituting this equality into Eq.(D.34) yields:

$$\begin{aligned} \pi_{P,t} + \frac{3}{4}\pi_{P,t}^2 + \frac{1-\alpha\delta}{2}\pi_{P,t}Z_t = {} & \kappa\left[\widetilde{kk}_{t,t} - \widetilde{ff}_{t,t} + \frac{1}{2}\left(\widetilde{kk}_{t,t}^2 - \widetilde{ff}_{t,t}^2\right)\right] \\ & + \delta \mathrm{E}_t \pi_{P,t+1} + \frac{(1-\alpha\delta)\delta}{2}\mathrm{E}_t \pi_{P,t+1}Z_{t+1} \\ & + \frac{3\delta}{4}\mathrm{E}_t\pi_{P,t+1}^2 + \frac{\delta\Theta}{4}\mathrm{E}_t\pi_{P,t+1}^2 \\ & + o\left(\|\xi\|^3\right). \end{aligned}$$

Adding $\frac{\Theta}{4}\pi_{P,t}^2$ to both sides in this equality, we have:

$$\mathcal{M}_t = \kappa\left[\widetilde{kk}_{t,t} - \widetilde{ff}_{t,t} + \frac{1}{2}\left(\widetilde{kk}_{t,t}^2 - \widetilde{ff}_{t,t}^2\right)\right] + \delta\mathrm{E}_t\mathcal{M}_{t+1} + \frac{\Theta}{4}\pi_{P,t}^2, \tag{D.40}$$

with $\mathcal{M}_t \equiv \pi_{P,t} + \frac{3}{4}\pi_{P,t}^2 + \frac{1-\alpha\delta}{2}\pi_{P,t}Z_t + \frac{\Theta}{4}\pi_{P,t}^2$. Substituting Eq.(D.38) into Eq.(D.40), we have:

$$\pi_{P,t} = \delta\pi_{P,t+1} + \frac{\lambda}{1-\sigma_G}\tilde{y}_t + o\left(\|\xi\|^2\right)$$

in the first order. Thus, Eq.(D.40) corresponds to the second-order approximated NKPC in Benigno and Woodford[10]. Iterating Eq.(D.40) forward, we have:

$$\mathcal{M} = \kappa \mathrm{E}_0 \sum_{t=0}^{\infty} \delta^t \left[\widetilde{kk}_{t,t} - \widetilde{ff}_{t,t} + \frac{1}{2} \left(\widetilde{kk}^2_{t,t} - \widetilde{ff}^2_{t,t} \right) \right] + \mathrm{E}_0 \sum_{t=0}^{\infty} \delta^t \frac{\Theta}{4} \pi^2_{P,t}, \quad \text{(D.41)}$$

with $\mathcal{M} \equiv \mathcal{M}_0$ where we take an expectation in period zero.

Eq.(D.28) implies:

$$\begin{aligned} \widetilde{kk}^2_{t,t} - \widetilde{ff}^2_{t,t} &= \tilde{\omega}_1 \tilde{y}_t^2 - \frac{2\sigma_G (1+\varphi)}{(1-\sigma_G)\lambda} \tilde{y}_t a_t + \frac{2\sigma_G (1+\varphi)}{(1-\sigma_G)\lambda} \tilde{y}_t g_t + \text{t.i.p.} \\ &= \tilde{\omega}_1 \left[y_t - \tilde{\omega}_4 \tilde{\omega}_2 a_t - \frac{\sigma_G}{\lambda} \tilde{\omega}_3 g_t \right]^2, \end{aligned}$$

with $\tilde{\omega}_1 \equiv \frac{\varsigma}{(1-\sigma_G)^2}$, $\tilde{\omega}_2 \equiv 1 + \frac{\sigma_G}{\varsigma}$, $\tilde{\omega}_3 \equiv 1 - \frac{(1-\sigma_G)(1+\varphi)}{\varsigma}$ and $\tilde{\omega}_4 \equiv \frac{(1-\sigma_G)(1+\varphi)}{\lambda}$. Substituting this equality and Eq.(D.38) into Eq.(D.41), we have:

$$\begin{aligned} \mathcal{M} &= \kappa \mathrm{E}_0 \sum_{t=0}^{\infty} \delta^t \left\{ \frac{\lambda}{1-\sigma_G} \tilde{y}_t + \frac{\tilde{\omega}_1}{2} \left[y_t - \tilde{\omega}_4 \tilde{\omega}_2 a_t - \frac{\sigma_G}{\lambda} \tilde{\omega}_3 g_t \right]^2 + \frac{\Theta}{4} \pi^2_{P,t} \right\} \\ &\quad + \text{t.i.p.} + o\left(\|\xi\|^3 \right). \end{aligned} \quad \text{(D.42)}$$

The counterpart of Eq.(D.42) in country F is derived similarly as:

$$\begin{aligned} \mathcal{M}^* &= \kappa \mathrm{E}_0 \sum_{t=0}^{\infty} \delta^t \left\{ \frac{\lambda}{1-\sigma_G} \tilde{y}^*_t + \frac{\tilde{\omega}_1}{2} \left[y^*_t - \tilde{\omega}_4 \tilde{\omega}_2 a^*_t - \frac{\sigma_G}{\lambda} \tilde{\omega}_3 g^*_t \right]^2 + \frac{\Theta}{4} \left(\pi^*_{P,t} \right)^2 \right\} \\ &\quad + \text{t.i.p.} + o\left(\|\xi\|^3 \right). \end{aligned}$$

Combining this equality and Eq.(D.42) yields:

$$\begin{aligned} \mathcal{M}^W &= \kappa \mathrm{E}_0 \sum_{t=0}^{\infty} \delta^t \left\{ \frac{\lambda}{1-\sigma_G} \tilde{y}^W_t + \frac{\tilde{\omega}_1}{4} \left[\left(y_t - \tilde{\omega}_4 \tilde{\omega}_2 a_t - \frac{\sigma_G}{\lambda} \tilde{\omega}_3 g_t \right)^2 + \left(y^*_t \right.\right.\right. \\ &\quad \left.\left.\left. - \tilde{\omega}_4 \tilde{\omega}_2 a^*_t - \frac{\sigma_G}{\lambda} \tilde{\omega}_3 g^*_t \right)^2 \right] + \frac{\Theta}{8} \left[\pi^2_{P,t} + \left(\pi^*_{P,t} \right)^2 \right] \right\} \\ &\quad + \text{t.i.p.} + o\left(\|\xi\|^3 \right). \end{aligned} \quad \text{(D.43)}$$

D.3 Step 3: Elimination of the Linear Term and Completing the Derivation

Multiplying Φ by both sides in Eq.(D.43), and subtracting this from Eq.(D.13), we have:

$$\mathcal{W}^W - \Phi \mathcal{M}^W = \mathrm{E}_0 \sum_{t=0}^{\infty} \delta^t \left\{ \frac{1}{(1-\sigma_G)4} \left[(1+\varphi)(1+\Phi) + \kappa \Phi \tilde{\omega}_1 \right] \left[y_t^2 + (y^*_t)^2 \right] \right.$$

$$
\begin{aligned}
&-(1+\varphi)\left[\frac{2\kappa\Phi\left(1-\sigma_G\right)}{\lambda}\tilde{\omega}_2+\frac{1+\Phi}{\left(1-\sigma_G\right)2}\right]\left(y_t a_t+y_t^* a_t^*\right) \\
&-\frac{2\kappa\Phi\sigma_G}{\lambda}\tilde{\omega}_3\left(y_t g_t+y_t^* g_t^*\right)+\frac{1}{4}\left[\frac{(1+\Phi)\theta}{\left(1-\sigma_G\right)\kappa}+\frac{\Phi\Theta}{2}\right]\left[\pi_{P,t}^2+\left(\pi_{P,t}^*\right)^2\right] \\
&\left.+\frac{\Phi(1-\kappa\lambda)}{1-\sigma_G}\tilde{y}_t^W\right\}+\text{t.i.p.}+o\left(\|\xi\|^3\right).
\end{aligned}
$$

Rearranging this equality, we have:

$$
\begin{aligned}
\mathcal{W}^W \quad = \quad & -\mathrm{E}_0\sum_{t=0}^{\infty}\delta^t\left\{\frac{1}{\left(1-\sigma_G\right)4}\left[(1+\varphi)(1+\Phi)+\kappa\Phi\tilde{\omega}_1\right]\left[y_t^2+\left(y_t^*\right)^2\right]\right. \\
&-(1+\varphi)\left[\frac{2\kappa\Phi\left(1-\sigma_G\right)}{\lambda}\tilde{\omega}_2+\frac{1+\Phi}{\left(1-\sigma_G\right)2}\right]\left(y_t a_t+y_t^* a_t^*\right) \\
&\left.-\frac{2\kappa\Phi\sigma_G}{\lambda}\tilde{\omega}_3\left(y_t g_t+y_t^* g_t^*\right)+\frac{1}{4}\left[\frac{(1+\Phi)\theta}{\left(1-\sigma_G\right)\kappa}+\frac{\Phi\Theta}{2}\right]\left[\pi_{P,t}^2+\left(\pi_{P,t}^*\right)^2\right]\right\} \\
&+\mathrm{E}_0\sum_{t=0}^{\infty}\delta^t\frac{\Phi(1-\kappa\lambda)}{1-\sigma_G}\tilde{y}_t^W+\Phi\mathcal{M}^W+\text{t.i.p.}+o\left(\|\xi\|^3\right). \qquad \text{(D.44)}
\end{aligned}
$$

Note that $\mathcal{M}^W=\mathrm{E}_0\sum_{t=0}^{\infty}\delta^t\frac{\kappa\lambda}{1-\sigma_G}y_t^W+o\left(\|\xi\|^2\right)$ because of Eq.(D.43). Thus, the second and the third terms in Eq.(D.44) can be rewritten as:

$$
\begin{aligned}
\mathrm{E}_0\sum_{t=0}^{\infty}\delta^t\frac{\Phi(1-\kappa\lambda)}{1-\sigma_G}\tilde{y}_t^W+\Phi\mathcal{M}^W \quad &= \quad \mathrm{E}_t\sum_{k=0}^{\infty}\delta^k\frac{\Phi(1-\kappa\lambda)}{1-\sigma_G}\tilde{y}_t^W+\Phi\mathcal{M}^W+\frac{\Phi}{\kappa\lambda}\mathcal{M}^W-\frac{\Phi}{\kappa\lambda}\mathcal{M}^W \\
&= \quad \mathrm{E}_0\sum_{t=0}^{\infty}\delta^t\frac{\Phi(1-\kappa\lambda)}{1-\sigma_G}\tilde{y}_t^W-\Phi\left(\frac{1}{\kappa\lambda}-1\right)\mathcal{M}^W+\frac{\Phi}{\kappa\lambda}\mathcal{M}^W \\
&= \quad \frac{\Phi}{\kappa\lambda}\mathcal{M}^W \\
&= \quad \Gamma_0. \qquad \text{(D.45)}
\end{aligned}
$$

We use this fact to derive the above equality as follows:

$$
\begin{aligned}
\pi_{P,0}^W \quad &= \quad \mathrm{E}_0\sum_{t=0}^{\infty}\delta^t\frac{\kappa\lambda}{1-\sigma_G}y_t^W+o\left(\|\xi\|^2\right), \\
&= \quad \mathcal{M}^W,
\end{aligned}
$$

which can be derived by iterating the second-order approximated period FONCs for firms, namely, $\pi_{P,t}=\delta\mathrm{E}_t\pi_{P,t+1}+\frac{\lambda}{1-\sigma_G}\tilde{y}_t+o\left(\|\xi\|^2\right)$.

Substituting Eq.(D.45) into Eq.(D.44) and rearranging, we have:

$$
\begin{aligned}
\mathcal{W}^W \quad = \quad & -\mathrm{E}_0\sum_{t=0}^{\infty}\delta^t\left\{\frac{1}{\left(1-\sigma_G\right)4}\omega_1\left[y_t^2+\left(y_t^*\right)^2\right]-\omega_2\left(y_t a_t+y_t^* a_t^*\right),\right. \\
&\left.-\omega_3\left(y_t g_t+y_t^* g_t^*\right)+\frac{1}{4}\omega_4\left[\pi_{P,t}^2+\left(\pi_{P,t}^*\right)^2\right]\right\}+\Gamma_0+\text{t.i.p.}+o\left(\|\xi\|^3\right),
\end{aligned}
$$

with $\omega_1 \equiv (1+\varphi)(1+\Phi) + \kappa\Phi\tilde{\omega}_1$, $\omega_2 \equiv (1+\varphi)\left[\frac{2\kappa\Phi(1-\sigma_G)\tilde{\omega}_2}{\lambda} + \frac{1+\Phi}{(1-\sigma_G)2}\right]$, $\omega_3 \equiv \frac{2\kappa\Phi\sigma_G\tilde{\omega}_3}{\lambda}$ and $\omega_4 \equiv \frac{(1+\Phi)\theta}{(1-\sigma_G)\kappa} + \frac{\Phi\Theta}{2}$. As shown in this equality, the linear term disappears. By arranging this equality, we have:

$$\begin{aligned}\mathcal{W}^W &= -\frac{1}{2}\mathrm{E}_0 \sum_{t=0}^{\infty} \delta^t \left[\frac{\Lambda_y}{2}\hat{y}_t^2 + \frac{\Lambda_y}{2}(\hat{y}_t^*)^2 + \frac{\Lambda_\pi}{2}\pi_{P,t}^2 + \frac{\Lambda_\pi}{2}\left(\pi_{P,t}^*\right)^2\right] + \Gamma_0 + \text{t.i.p.} \\ &\quad + o\left(\|\xi\|^3\right),\end{aligned}$$

which corresponds to the second-order approximated welfare function in the text.

D.4 Other Details on Coefficients and the NKPC in Terms of the Welfare-relevant Output Gap

Note that complicated coefficients associated with the target level of output are as follows:

$$\begin{aligned}\Omega_1 &\equiv \frac{(1-\sigma_G)(1+\varphi)\left[4\kappa\Phi(1-\sigma_G)^2(\varsigma+\sigma_G) + \lambda\varsigma(1+\Phi)\right]}{(\chi+\kappa\Phi+\varsigma)\lambda\varsigma} \\ \Omega_2 &\equiv \frac{(1-\sigma_G)^2 4\kappa\Phi\sigma_G\left[\varsigma - (1-\sigma_G)(1+\varphi)\right]}{(\chi+\kappa\Phi\varsigma)\lambda\varsigma}.\end{aligned}$$

The NKPCs in terms of the welfare-relevant output gap are different to the NKPCs in terms of the output gap. Eq.(38) can be rewritten as:

$$\begin{aligned}\pi_{P,t} &= \delta\mathrm{E}_t\pi_{P,t+1} + \frac{\kappa\lambda}{1-\sigma_G}\tilde{y}_t \\ &= \delta\mathrm{E}_t\pi_{P,t+1} + \frac{\kappa\lambda}{1-\sigma_G}\left(\hat{y}_t + y_t^e - \bar{y}_t\right) \\ &= \delta\mathrm{E}_t\pi_{P,t+1} + \frac{\kappa\lambda}{1-\sigma_G}\hat{y}_t + \frac{\kappa\lambda}{1-\sigma_G}\left(\Omega_1 - \bar{\beta}\right)a_t + \frac{\kappa\lambda}{1-\sigma_G}\left(\Omega_2 - \frac{\sigma_G}{\lambda}\right)g_t \\ &= \delta\mathrm{E}_t\pi_{P,t+1} + \frac{\kappa\lambda}{1-\sigma_G}\hat{y}_t + \kappa(1+\varphi)\Omega_3 a_t + \kappa\sigma_G\Omega_4 g_t \\ &= \delta\mathrm{E}_t\pi_{P,t+1} + \frac{\kappa\lambda}{1-\sigma_G}\hat{y}_t + \varepsilon_t, \qquad \text{(D.46)}\end{aligned}$$

with $\Omega_3 \equiv \frac{4\kappa\Phi(1-\sigma_G)^2(\varsigma+\sigma_G)+\lambda\varsigma(1+\Phi)}{(\chi+\kappa\Phi+\varsigma)\varsigma}$ and $\Omega_4 \equiv \frac{(1-\sigma_G)4\kappa\Phi[\varsigma-(1-\sigma_G)(1+\varphi])}{(\chi+\kappa\Phi+\varsigma)\varsigma} - \frac{1}{1-\sigma_G}$. This equality corresponds to Eq.(21) in the text.

E Lagrangian and its FONCs

E.1 Optimal Monetary Policy Alone

The Lagrangian is given by:

$$\begin{aligned}
\pounds \;=\; & \mathrm{E}_0 \left\{ \sum_{t=0}^{\infty} \delta^t \left[L_t^W + \mu_{1,t} \left(\tilde{y}_t^W - \frac{\beta_W}{1-\sigma_G} \tilde{y}_{t+1}^W + \beta_W \hat{r}_t - \beta_W \pi_{t+1}^W - \frac{\beta_W}{\delta} \hat{r}_{t-1} \right.\right.\right. \\
& \left. + \frac{\beta_W}{\delta} \pi_t^W + \beta_W b_t^W \right) + \mu_{2,t} \left[\tilde{y}_t^R - \beta_R (1-\gamma) \upsilon \mathsf{n}_t + \beta_R (1-\gamma) \mathsf{n}_{t-1} \right] \\
& + \mu_{3,t} \left(\pi_{P,t} - \delta \pi_{P,t+1} - \frac{\kappa\lambda}{1-\sigma_G} \tilde{y}_t \right) + \mu_{4,t} \left(\pi_{P,t}^* - \delta \pi_{P,t+1}^* - \frac{\kappa\lambda}{1-\sigma_G} \tilde{y}_t^* \right) \\
& \left.\left. + \mu_{5,t} \left(\mathsf{n}_t - \frac{\delta}{1+\delta+\kappa} \mathsf{n}_{t+1} + \frac{\kappa\varphi}{1+\delta+\kappa} \tilde{y}_t^R - \frac{1}{1+\delta+\kappa} \mathsf{n}_{t-1} \right) \right] \right\},
\end{aligned}$$

because $b_t = b_t^* = 0$ for all t.

The FONCs are as follows:

$$\begin{aligned}
\frac{\Lambda_\pi}{2} \pi_{P,t} + \frac{\beta_W}{2\delta} \left(\mu_{1,t} - \mu_{1,t-1} \right) + \left(\mu_{3,t} - \mu_{3,t-1} \right) &= 0 \\
\frac{\Lambda_\pi}{2} \pi_{P,t}^* + \frac{\beta_W}{2\delta} \left(\mu_{1,t} - \mu_{1,t-1} \right) + \left(\mu_{4,t} - \mu_{4,t-1} \right) &= 0 \\
\frac{\Lambda_y}{2} \hat{y}_t + \frac{1}{2} \mu_{1,t} + \mu_{2,t} - \frac{\lambda\kappa}{(1-\sigma_G)} \mu_{3,t} + \frac{\kappa\varphi}{1+\delta+\kappa} \mu_{5,t} & \\
- \frac{\beta_W}{(1-\sigma_G) 2\delta} \mu_{1,t-1} &= 0 \\
\frac{\Lambda_y}{2} \tilde{y}_t^* + \frac{1}{2} \mu_{1,t} - \mu_{2,t} - \frac{\lambda\kappa}{(1-\sigma_G)} \mu_{4,t} - \frac{\kappa\varphi}{1+\delta+\kappa} \mu_{5,t} & \\
- \frac{\beta_W}{(1-\sigma_G) 2\delta} \mu_{1,t-1} &= 0 \\
- \beta_R (1-\gamma) \upsilon \mu_{2,t} - \mu_{5,t} - \frac{1}{1+\delta+\kappa} \mu_{5,t-1} &= 0 \\
\mu_{1,t} &= 0 \qquad \text{(E.1)}
\end{aligned}$$

Note that the fifth equality in Eq.(E.1) corresponds to the second equality in Eq.(24) in the text. Because of commitment, a lagged Lagrangian multiplier appears.

Combining the first to the fourth and the sixth equalities in Eq.(E.1), we have:

$$\begin{aligned}
\Lambda_\pi \pi_t^W + \left(\mu_{3,t} - \mu_{3,t-1} \right) + \left(\mu_{4,t} - \mu_{4,t-1} \right) &= 0, \\
\frac{1+\varphi}{1-\sigma_G} \hat{y}_t^W - \frac{\kappa\lambda}{(1-\sigma_G) 2} \mu_{3,t} - \frac{\kappa\lambda}{(1-\sigma_G) 2} \mu_{4,t} &= 0. \qquad \text{(E.2)}
\end{aligned}$$

Combining both equalities in Eq.(E.2) yields:

$$\pi_t^W = - \frac{\Lambda_y (1-\sigma_G)}{\Lambda_\pi \kappa\lambda} \left(\hat{y}_t^W - \hat{y}_{t-1}^W \right),$$

which is Eq.(22) in the text.

Combining the first to the fourth and the sixth equalities in Eq.(E.1), we have:

$$\begin{aligned} \frac{\Lambda_\pi}{2}\pi^R_{P,t} + (\mu_{3,t} - \mu_{3,t-1}) - (\mu_{4,t} - \mu_{4,t-1}) &= 0, \\ \frac{\Lambda_y}{2}\hat{y}^R_t + \mu_{2,t} - \frac{\kappa\lambda}{1-\sigma_G}\mu_{3,t} + \frac{\kappa\lambda}{1-\sigma_G}\mu_{4,t} + \frac{2\kappa\varphi}{1+\delta+\kappa}\mu_{5,t} &= 0. \end{aligned} \tag{E.3}$$

Combining both equalities in Eq.(E.3) yields:

$$\begin{aligned} \pi^R_{P,t} &= -\frac{\Lambda_y(1-\sigma_G)}{\Lambda_\pi\kappa\lambda}\left(\hat{y}^R_t - \hat{y}^R_{t-1}\right) - \frac{2(1-\sigma_G)}{\Lambda_\pi\kappa\lambda}\left(\mu_{2,t} - \mu_{2,t-1}\right) \\ &\quad - \frac{(1-\sigma_G)4\kappa\varphi}{\Lambda_\pi\kappa\lambda(1+\delta+\kappa)}\left(\mu_{5,t} - \mu_{5,t-1}\right), \end{aligned} \tag{E.4}$$

which is the first equality in Eq.(24) in the text.

E.2 Optimal Monetary and Fiscal Policy

The Lagrangian is given by:

$$\begin{aligned} \pounds &= \mathrm{E}_0\left\{\sum_{t=0}^{\infty}\delta^t\left[L^W_t + \mu_{1,t}\left(\tilde{y}^W_t - \frac{\beta_W}{1-\sigma_G}\tilde{y}^W_{t+1} + \beta_W\hat{r}_t - \beta_W\pi^W_{t+1} - \frac{\beta_W}{\delta}\hat{r}_{t-1}\right.\right.\right. \\ &\quad \left. + \frac{\beta_W}{\delta}\pi^W_t + \beta_W b^W_t - \frac{\beta_W}{\delta}b^W_{t-1}\right) + \mu_{2,t}\left[\tilde{y}^R_t + \beta_R\delta b^R_t - \beta_R(1-\gamma)\upsilon\mathsf{n}_t\right. \\ &\quad \left. + \beta_R(1-\gamma)\mathsf{n}_{t-1}\right] + \mu_{3,t}\left(\pi_{P,t} - \delta\pi_{P,t+1} - \frac{\kappa\lambda}{1-\sigma_G}\tilde{y}_t\right) + \mu_{4,t}\left(\pi^*_{P,t}\right. \\ &\quad \left. - \delta\pi^*_{P,t+1} - \frac{\kappa\lambda}{1-\sigma_G}\tilde{y}^*_t\right) + \mu_{5,t}\left(\mathsf{n}_t - \frac{\delta}{1+\delta+\kappa}\mathsf{n}_{t+1} + \frac{\kappa\varphi}{1+\delta+\kappa}\tilde{y}^R_t\right. \\ &\quad \left.\left.\left. - \frac{1}{1+\delta+\kappa}\mathsf{n}_{t-1}\right)\right]\right\}. \end{aligned}$$

The FONCs of the Lagrangian are given by Eq.(E.1) and the following equalities:

$$\begin{aligned} \frac{\beta_W}{2}\mu_{1,t} + \beta_R\delta\mu_{2,t} &= 0, \\ \frac{\beta_W}{2}\mu_{1,t} - \beta_R\delta\mu_{2,t} &= 0. \end{aligned} \tag{E.5}$$

Combining both equalities in Eq.(E.5), we have:

$$\mu_{2,t} = 0. \tag{E.6}$$

Substituting Eq.(E.6) into Eq.(E.4), we have:

$$\pi^R_{P,t} = -\frac{\Lambda_y(1-\sigma_G)}{\Lambda_\pi\kappa\lambda}\left(\hat{y}^R_t - \hat{y}^R_{t-1}\right) - \frac{(1-\sigma_G)4\kappa\varphi}{\Lambda_\pi\kappa\lambda(1+\delta+\kappa)}\left(\mu_{5,t} - \mu_{5,t-1}\right). \tag{E.7}$$

Substituting Eq.(E.6) and the initial condition $\mu_{5,-1} = 0$ into the fifth equality in Eq.(E.1), we have:

$$\mu_{5,t} = 0. \tag{E.8}$$

Substituting Eq.(E.8) into Eq.(E.7) yields:

$$\pi^R_{P,t} = -\frac{\Lambda_y (1 - \sigma_G)}{\Lambda_\pi \kappa \lambda} \left(\hat{y}^R_t - \hat{y}^R_{t-1} \right),$$

which is Eq.(25) in the text.

F Derivation of Social Loss

Using the stable roots obtained by analyzing the determinacy, this section calculates social loss analytically.[33] We assume that the model includes the price shocks that forbid the central bank from being able to stabilize inflation and the output gap simultaneously.

Similar to Eq.(D.46), we have the NKPC in terms of the welfare-relevant output gap in country F as follows:

$$\pi^*_{P,t} = \delta \mathrm{E}_t \pi_{P,t+1} + \frac{\kappa \lambda}{1 - \sigma_G} \hat{y}^*_t + \varepsilon^*_t, \tag{F.1}$$

with $\varepsilon^*_t \equiv \kappa (1 + \varphi) \Omega_3 a^*_t + \kappa \sigma_G \Omega_4 g^*_t$. Combining this equality and Eq.(D.46), we have:

$$\pi^W_t = \delta \mathrm{E}_t \pi^W_{t+1} + \frac{\kappa \lambda}{1 - \sigma_G} \hat{y}^W_t + \varepsilon^W_t.$$

Note that $\varepsilon^W_t = \Omega_5 \gamma \xi_{H,t} + \Omega_5 \gamma \xi_{F,t} + \Omega_5 (1 - \gamma) \xi_{\mathcal{N},t} + \Omega_5 (1 - \gamma) \xi^*_{\mathcal{N},t} - \Omega_6 \xi^W_{G,t}$ with $\Omega_5 \equiv \frac{\kappa(1+\varphi)\Omega_3}{2}$ and $\Omega_6 \equiv \kappa \sigma_G \Omega_4$.

Firstly, we calculate the system of the average block. Substituting Eq.(22) in the text into this equality, we have:

$$\mathrm{E}_t \hat{y}^W_{t+1} = \Omega_7 \delta^{-1} \hat{y}^W_t - \delta^{-1} \hat{y}^W_{t-1} + \Omega_8 \delta^{-1} \varepsilon^W_t, \tag{F.2}$$

with $\Omega_7 \equiv \frac{1-\sigma_G}{\Lambda_\pi \kappa \lambda} \left(1 + \delta + \frac{\Omega_8 \kappa \lambda}{1 - \sigma_G} \right)$ and $\Omega_8 \equiv \frac{\Lambda_\pi \kappa \lambda}{\Lambda_y (1 - \sigma_G)}$. Its vector form is given by:

$$\begin{bmatrix} \mathrm{E}_t \hat{y}^W_{t+1} \\ \hat{y}^W_t \end{bmatrix} = \mathbf{M} \begin{bmatrix} \hat{y}^W_t \\ \hat{y}^W_{t-1} \end{bmatrix} + \begin{bmatrix} \Omega_8 \delta^{-1} \\ 0 \end{bmatrix} \varepsilon^W_t$$

with $\mathbf{M} \equiv \begin{bmatrix} \Omega_7 \delta^{-1} & -\delta^{-1} \\ 1 & 0 \end{bmatrix}$.

[33] See Monacelli[25] and Walsh[34].

These roots are the solution to the characteristic equations as follows:

$$\Psi^2 - \mathrm{tr}\left(\mathbf{M}\right) + \det\left(\mathbf{M}\right) = 0,$$

with $\mathrm{tr}\left(\mathbf{M}\right) = \Omega_7\delta^{-1} = \Psi_1 + \Psi_2$ and $\det\left(\mathbf{M}\right) = \delta^{-1} = \Psi_1\Psi_2 > 1$.

Let us suppose $\mid \Psi_1 \mid < 1$. Ψ_2 satisfies:

$$\Psi_1 < 1 < \delta^{-1} < \Psi_2 = \delta^{-1}\Psi_1^{-1}.$$

The pair of solutions to the characteristic equation is as follows:

$$\Psi_{1,2} = \frac{\Omega_7}{2\delta}\left(1 \pm \sqrt{1 - \frac{4\delta}{\Omega_7^2}}\right),$$

with $\Psi_{1,2}$ denoting the pair of solutions to the characteristic equation.

Eq.(F.2) can be rewritten as:

$$\left(1 - \frac{\Omega_7}{\delta}\mathrm{L} + \frac{1}{\delta}\mathrm{L}^2\right)\hat{y}_t^W = \frac{\Omega_8}{\delta}\varepsilon_{t-1}^W, \tag{F.3}$$

where L is the lag operator. The coefficient on the LHS in Eq.(F.3) can be rewritten as:

$$1 - \frac{\Omega_7}{\delta}\mathrm{L} + \frac{1}{\delta}\mathrm{L}^2 = \left(1 - \Psi_1\mathrm{L}\right)\left(1 - \Psi_2\mathrm{L}\right).$$

Substituting this into Eq.(F.3), we have:

$$\left(1 - \Psi_1\mathrm{L}\right)\left(1 - \Psi_2\mathrm{L}\right)\hat{y}_t^W = \frac{\Omega_8}{\delta}\varepsilon_t^W. \tag{F.4}$$

Because $\left(1 - \Psi_2\mathrm{L}\right)^{-1} = -\sum_{k=1}^{\infty}\left(\Psi_2\mathrm{L}\right)^{-k}$, this can be rewritten as:

$$\left(1 - \Psi_1\mathrm{L}\right)\hat{y}_t^W = -\Omega_8\Psi_1\varepsilon_t^W,$$

where we use the fact that $\Psi_2 = \delta^{-1}\Psi_1^{-1}$. Thus, the final form of the solution is given by:

$$\hat{y}_t^W = -\Omega_8\Psi_1\sum_{k=0}^{\infty}\Psi_1^k\varepsilon_{t-k}^W. \tag{F.5}$$

Secondly, we calculate the system of the relative block. Subtracting its counterpart in country F from Eq.(D.46), we have:

$$\pi_t^R = \delta\mathrm{E}_t\pi_{t+1}^R + \frac{\kappa\lambda}{1 - \sigma_G}\hat{y}_t^R + \varepsilon_t^R,$$

with $\varepsilon_t^R = 2\Omega_5\gamma\xi_{H,t} - 2\Omega_5\gamma\xi_{F,t} + 2\Omega_5\left(1 - \gamma\right)\xi_{\mathcal{N},t} - 2\Omega_5\left(1 - \gamma\right)\xi_{\mathcal{N},t}^* - \Omega_6\xi_{G,t}^R$. Substituting Eq.(25) in the text into this equality and using a similar procedure to derive Eq.(F.5), we have:

$$\hat{y}_t^R = -\Omega_8\Psi_1\sum_{k=0}^{\infty}\Psi_1^k\varepsilon_{t-k}^R. \tag{F.6}$$

Note that $v_t = v_t^W + \frac{1}{2}v_t^R$. Thus, combining Eqs.(F.5) and (F.6), we have:

$$\hat{y}_t = -\Omega_8 \Psi_1 \sum_{k=0}^{\infty} \Psi_1^k \varepsilon_{t-k}. \tag{F.7}$$

Subtracting Eq.(F.7) with a one period lag from Eq.(F.7), we have:

$$(\hat{y}_t - \hat{y}_{t-1}) = \Omega_8 \Psi_1 (1 - \Psi_1) \sum_{k=0}^{\infty} \Psi_1^{k-1} \varepsilon_{t-k} - \Omega_8 \varepsilon_t. \tag{F.8}$$

The FONCs under the optimal monetary and policy regime imply:

$$\pi_{P,t} = -\Omega_8^{-1} (\hat{y}_t - \hat{y}_{t-1}). \tag{F.9}$$

Combining Eqs.(F.7) and (F.8), we have:

$$\pi_{P,t} = -\left[\Psi_1 (1 - \Psi_1) \sum_{k=0}^{\infty} \Psi_1^{k-1} \varepsilon_{t-k} - \varepsilon_t\right]. \tag{F.10}$$

Note that $v_t^* = v_t^W - \frac{1}{2}v_t^R$. Thus, combining Eqs.(F.5) and (F.6), we have:

$$\hat{y}_t^* = -\Omega_8 \Psi_1 \sum_{k=0}^{\infty} \Psi_1^k \varepsilon_{t-k}^*. \tag{F.11}$$

Subtracting Eq.(F.7) with a one period lag from Eq.(F.7) yields:

$$\left(\hat{y}_t^* - \hat{y}_{t-1}^*\right) = \Omega_8 \Psi_1 (1 - \Psi_1) \sum_{k=0}^{\infty} \Psi_1^{k-1} \varepsilon_{t-k}^* - \Omega_8 \varepsilon_t^*. \tag{F.12}$$

The FONCs under the optimal monetary and policy regime imply:

$$\pi_{P,t}^* = -\Omega_8^{-1} \left(\hat{y}_t^* - \hat{y}_{t-1}^*\right). \tag{F.13}$$

Combining Eqs.(F.7) and (F.8), we have:

$$\pi_{P,t}^* = -\left[\Psi_1 (1 - \Psi_1) \sum_{k=0}^{\infty} \Psi_1^{k-1} \varepsilon_{t-k}^* - \varepsilon_t^*\right]. \tag{F.14}$$

Eq.(F.7) implies:

$$\hat{y}_t^2 = (\Omega_8 \Psi_1)^2 \sum_{k=0}^{\infty} \Psi_1^{2k} \varepsilon_{t-k}^2. \tag{F.15}$$

Eq.(F.10) implies:

$$(\pi_{P,t} - \varepsilon_t)^2 = \Psi_1^2 (1 - \Psi_1)^2 \sum_{k=0}^{\infty} \Psi_1^{2(k-1)} \varepsilon_{t-k}^2. \tag{F.16}$$

The LHS in Eq.(F.16) can be rewritten as:

$$(\pi_{P,t}-\varepsilon_t)^2=\pi_{P,t}^2-2\pi_{P,t}\varepsilon_t+\varepsilon_t^2. \tag{F.17}$$

Multiplying by ε_t on both sides of Eq.(F.10), we have:

$$\pi_{P,t}\varepsilon_t=\Psi_1\varepsilon_t^2, \tag{F.18}$$

because the serial correlation of shocks is zero. Combining Eqs.(F.17) and (F.18) yields:

$$(\pi_{P,t}-\varepsilon_t)^2=\pi_{P,t}^2+(1-2\Psi_1)\,\varepsilon_t^2. \tag{F.19}$$

Combining Eqs.(F.16) and (F.19), we have:

$$\pi_{P,t}^2=\Psi_1^2(1-\Psi_1)^2\sum_{k=0}^{\infty}\Psi_1^{2(k-1)}\varepsilon_{t-k}^2-(1-2\Psi_1)\,\varepsilon_t^2. \tag{F.20}$$

By using a similar procedure to derive Eqs.(F.15) and (F.20), we have:

$$\begin{aligned}(\hat{y}_t^*)^2 &= (\Omega_8\Psi_1)^2\sum_{k=0}^{\infty}\Psi_1^{2k}\left(\varepsilon_{t-k}^*\right)^2, && \text{(F.21)}\\ \left(\pi_{P,t}^*\right)^2 &= \Psi_1^2(1-\Psi_1)^2\sum_{k=0}^{\infty}\Psi_1^{2(k-1)}\left(\varepsilon_{t-k}^*\right)^2-(1-2\Psi_1)\,(\varepsilon_t^*)^2. && \text{(F.22)}\end{aligned}$$

Under the self-oriented setting, the Lagrangian for country H is given by:

$$\begin{aligned}\pounds \;=\; & \mathrm{E}_0\left\{\sum_{t=0}^{\infty}\delta^t\left[L_t+\mu_{1,t}\left(\tilde{y}_t^W-\frac{\beta_W}{1-\sigma_G}\tilde{y}_{t+1}^W+\beta_W\hat{r}_t-\beta_W\pi_{t+1}^W-\frac{\beta_W}{\delta}\hat{r}_{t-1}\right.\right.\right.\\ & \left.+\frac{\beta_W}{\delta}\pi_t^W+\beta_W b_t^W-\frac{\beta_W}{\delta}b_{t-1}^W\right)+\mu_{2,t}\left[\tilde{y}_t^R+\beta_R\delta b_t^R-\beta_R(1-\gamma)\,\upsilon\mathsf{n}_t\right.\\ & \left.+\beta_R(1-\gamma)\,\mathsf{n}_{t-1}\right]+\mu_{3,t}\left(\pi_{P,t}-\delta\pi_{P,t+1}-\frac{\kappa\lambda}{1-\sigma_G}\tilde{y}_t\right)+\mu_{4,t}\left(\pi_{P,t}^*\right.\\ & \left.-\delta\pi_{P,t+1}^*-\frac{\kappa\lambda}{1-\sigma_G}\tilde{y}_t^*\right)+\mu_{5,t}\left(\mathsf{n}_t-\frac{\delta}{1+\delta+\kappa}\mathsf{n}_{t+1}+\frac{\kappa\varphi}{1+\delta+\kappa}\tilde{y}_t^R\right.\\ & \left.\left.-\frac{1}{1+\delta+\kappa}\mathsf{n}_{t-1}\right)\right]+\mu_{6,t}\left[\pi_t^W+\frac{(1-\sigma_G)\,\Lambda_y}{\kappa\lambda\Lambda_\pi}\hat{y}_t^W-\frac{(1-\sigma_G)\,\Lambda_y}{\kappa\lambda\Lambda_\pi}\hat{y}_{t-1}^W\right]\\ & +\mu_{7,t}\left[\pi_{P,t}^R+\frac{(1-\sigma_G)\,\Lambda_y}{\kappa\lambda\Lambda_\pi}\hat{y}_t^R-\frac{(1-\sigma_G)\,\Lambda_y}{\kappa\lambda\Lambda_\pi}\hat{y}_{t-1}^R+\frac{2(1-\sigma_G)}{\Lambda_\pi\kappa\lambda}\mu_{2,t}\right.\\ & \left.-\frac{2(1-\sigma_G)}{\Lambda_\pi\kappa\lambda}\mu_{2,t-1}+\frac{4(1-\sigma_G)\,\kappa\varphi}{\Lambda_\pi\kappa\lambda(1+\delta+\kappa)}\mu_{5,t}-\frac{4(1-\sigma_G)\,\kappa\varphi}{\Lambda_\pi\kappa\lambda(1+\delta+\kappa)}\mu_{5,t-1}\right]\\ & \left.+\mu_{8,t}\left[\mu_{5,t}-(1-\gamma)\,\beta_R\upsilon\mu_{2,t}-\frac{1}{1+\delta+\kappa}\mu_{5,t-1}\right]\right\}.\end{aligned}$$

Because the central bank conducts optimal monetary policy, the FONCs under the optimal monetary policy alone, Eq.(24) in the text, appear in this Lagrangian as constraints. A similar Lagrangian is given for the government in country F although L_t^* replaces L_t. Note that any exogenous shifters are omitted in this Lagrangian.

The government in country H chooses the sequence $\{\pi_{P,t}, \hat{y}_t, \mathsf{n}_t, b_t\}_{t=0}^{\infty}$ while the government in country F chooses the sequence $\left\{\pi_{P,t}^*, \hat{y}_t^*, \mathsf{n}_t, b_t^*\right\}_{t=0}^{\infty}$ under commitment. The FONCs are given by:

$$\begin{aligned}
\Lambda_\pi \pi_{P,t} + \frac{\beta_W}{2\delta}\mu_{1,t} + \mu_{3,t} - \frac{\beta_W}{2\delta}\mu_{1,t-1} - \mu_{3,t-1} + \frac{1}{2}\mu_{6,t} + \mu_{7,t} &= 0 \\
\Lambda_y \hat{y}_t + \frac{1}{2}\mu_{1,t} + \mu_{2,t} - \frac{\kappa\lambda}{1-\sigma_G}\mu_{3,t} + \frac{\kappa\varphi}{1+\delta+\kappa}\mu_{5,t} - \frac{\beta_W}{(1-\sigma_G)\,2\delta}\mu_{1,t-1} & \\
+\frac{(1-\sigma_G)\,\Lambda_y}{\kappa\lambda\Lambda_\pi 2}\mu_{6,t} + \frac{(1-\sigma_G)\,\Lambda_y}{\kappa\lambda\Lambda_\pi}\mu_{7,t} &= 0 \\
-\beta_R(1-\gamma)\,\upsilon\mu_{2,t} + \mu_{5,t} - \frac{1}{1+\delta+\kappa}\mu_{5,t-1} &= 0 \\
\frac{\beta_W}{2}\mu_{1,t} + \beta_R\delta\mu_{2,t} &= 0 \\
\Lambda_\pi \pi_{P,t}^* + \frac{\beta_W}{2\delta}\mu_{1,t} + \mu_{4,t} - \frac{\beta_W}{2\delta}\mu_{1,t-1} - \mu_{4,t-1} + \frac{1}{2}\mu_{6,t} - \mu_{7,t} &= 0 \\
\Lambda_y \hat{y}_t^* + \frac{1}{2}\mu_{1,t} + \mu_{2,t} - \frac{\kappa\lambda}{1-\sigma_G}\mu_{4,t} - \frac{\kappa\varphi}{1+\delta+\kappa}\mu_{5,t} - \frac{\beta_W}{(1-\sigma_G)\,2\delta}\mu_{1,t-1} & \\
+\frac{(1-\sigma_G)\,\Lambda_y}{\kappa\lambda\Lambda_\pi 2}\mu_{6,t} - \frac{(1-\sigma_G)\,\Lambda_y}{\kappa\lambda\Lambda_\pi}\mu_{7,t} &= 0 \\
\frac{\beta_W}{2}\mu_{1,t} - \beta_R\delta\mu_{2,t} &= 0.
\end{aligned} \tag{F.23}$$

The fourth and the seventh equalities in Eq.(F.23) imply:

$$\mu_{1,t} = 0\ ;\ \mu_{2,t} = 0. \tag{F.24}$$

The third equality in Eq.(F.23) and Eq.(F.24) imply:

$$\mu_{5,t} = 0, \tag{F.25}$$

given the initial condition $\mu_{5,-1} = 0$. Substituting Eqs.(F.24) and (F.25) into the first, the second, the fifth and the sixth equalities in Eq.(F.23) yields:

$$\Lambda_\pi \pi_{P,t} + (\mu_{3,t} - \mu_{3,t-1}) + \frac{1}{2}\mu_{6,t} + \mu_{7,t} = 0 \tag{F.26}$$

$$\Lambda_y \hat{y}_t - \frac{\kappa\lambda}{1-\sigma_G}\mu_{3,t} + \frac{(1-\sigma_G)\,\Lambda_y}{\kappa\lambda\Lambda_\pi 2}\mu_{6,t} + \frac{(1-\sigma_G)\,\Lambda_y}{\kappa\lambda\Lambda_\pi}\mu_{7,t} = 0 \tag{F.27}$$

$$\Lambda_\pi \pi_{P,t}^* + (\mu_{4,t} - \mu_{4,t-1}) + \frac{1}{2}\mu_{6,t} - \mu_{7,t} = 0 \tag{F.28}$$

$$\Lambda_y \hat{y}_t^* - \frac{\kappa\lambda}{1-\sigma_G}\mu_{4,t} + \frac{(1-\sigma_G)\,\Lambda_y}{\kappa\lambda\Lambda_\pi 2}\mu_{6,t} - \frac{(1-\sigma_G)\,\Lambda_y}{\kappa\lambda\Lambda_\pi}\mu_{7,t} = 0 \tag{F.29}$$

Eqs.(F.26) and (F.28) imply the following:

$$\frac{1}{2}\mu_{6,t} = -\Lambda_\pi \pi_t^W - \frac{1}{2}(\mu_{3,t} - \mu_{3,t-1}) - \frac{1}{2}(\mu_{4,t} - \mu_{4,t-1}) \tag{F.30}$$

$$\mu_{7,t} = -\frac{\Lambda_\pi}{2}\pi_{P,t}^R - \frac{1}{2}(\mu_{3,t} - \mu_{3,t-1}) + \frac{1}{2}(\mu_{4,t} - \mu_{4,t-1}) \tag{F.31}$$

Combining Eq.(F.30) and Eq.(22) in the text, we have:

$$\frac{1}{2}\mu_{6,t} = \frac{(1-\sigma_G)\Lambda_y}{\kappa\lambda}\left(\hat{y}_t^W - \hat{y}_{t-1}^W\right) - \frac{1}{2}(\mu_{3,t} - \mu_{3,t-1}) - \frac{1}{2}\mu_{4,t}. \tag{F.32}$$

Combining Eqs.(F.27) and (F.29), we have:

$$-\frac{1}{2}(\mu_{3,t} - \mu_{3,t-1}) - \frac{1}{2}(\mu_{4,t} - \mu_{4,t-1}) = -\frac{(1-\sigma_G)\Lambda_y}{\kappa\lambda}\left(\hat{y}_t^W - y_{t-1}^W\right) - \frac{(1-\sigma_G)^2\Lambda_y}{(\kappa\lambda)^2\Lambda_\pi}(\mu_{6,t} - \mu_{6,t-1}).$$

Substituting this equality into Eq.(F.32) yields:

$$\left[\frac{1}{2} + \frac{(1-\sigma_G)^2\Lambda_y}{(\kappa\lambda)^2\Lambda_\pi}\right]\mu_{6,t} = \frac{(1-\sigma_G)^2\Lambda_y}{(\kappa\lambda)^2\Lambda_\pi}\mu_{6,t-1}.$$

This equality implies the following:

$$\mu_{6,t} = 0, \tag{F.33}$$

given the initial condition $\mu_{6,-1} = 0$.

Substituting Eqs.(F.24) and (F.25) into Eq.(24) in the text, we have:

$$\pi_{P,t}^R = -\frac{(1-\sigma_G)\Lambda_y}{\kappa\lambda\Lambda_\pi}\hat{y}_t^R + \frac{(1-\sigma_G)\Lambda_y}{\kappa\lambda\Lambda_\pi}\hat{y}_{t-1}^R.$$

Combining this equality and Eq.(F.31) yields:

$$\mu_{7,t} = \frac{(1-\sigma_G)\Lambda_y}{\kappa\lambda 2}\left(\hat{y}_t^R - \hat{y}_{t-1}^R\right) - \frac{1}{2}(\mu_{3,t} - \mu_{3,t-1}) + \frac{1}{2}(\mu_{4,t} - \mu_{4,t-1}). \tag{F.34}$$

Combining Eqs.(F.27) and (F.29), we have:

$$-\frac{1}{2}(\mu_{3,t} - \mu_{3,t-1}) + \frac{1}{2}(\mu_{4,t} - \mu_{4,t-1}) = -\frac{(1-\sigma_G)\Lambda_y}{2\kappa\mu_{4,t}\lambda}\left(\hat{y}_t^R - \hat{y}_{t-1}^R\right) - \frac{(1-\sigma_G)^2\Lambda_y}{\kappa\lambda\Lambda_\pi}(\mu_{7,t} - \mu_{7,t-1}).$$

Combining this equality and Eq.(F.34), we have:

$$\mu_{7,t} = -\frac{(1-\sigma_G)^2\Lambda_y}{\kappa\lambda\Lambda_\pi}(\mu_{7,t} - \mu_{7,t-1}),$$

which implies the following

$$\mu_{7,t} = 0, \tag{F.35}$$

given the initial condition $\mu_{7,-1} = 0$.

Substituting Eqs.(F.33) and (F.35) into Eqs.(F.26)–(F.29) yields:

$$\begin{aligned}
\pi_{P,t} &= -\frac{1}{\Lambda_\pi}\left(\mu_{3,t} - \mu_{3,t-1}\right) \\
\left(\mu_{3,t} - \mu_{3,t-1}\right) &= \frac{\left(1-\sigma_G\right)\Lambda_y}{\kappa\lambda}\left(\hat{y}_t - \hat{y}_{t-1}\right) \\
\pi^*_{P,t} &= -\frac{1}{\Lambda_\pi}\left(\mu_{4,t} - \mu_{4,t-1}\right) \\
\left(\mu_{4,t} - \mu_{4,t-1}\right) &= \frac{\left(1-\sigma_G\right)\Lambda_y}{\kappa\lambda}\left(\hat{y}^*_t - \hat{y}^*_{t-1}\right).
\end{aligned} \quad (F.36)$$

Combining the first and the second equalities in Eq.(F.36) yields:

$$\pi_{P,t} = -\frac{\left(1-\sigma_G\right)\Lambda_y}{\kappa\lambda\Lambda_\pi}\left(\hat{y}_t - \hat{y}_{t-1}\right). \quad (F.37)$$

Combining the third and the fourth equalities in Eq.(F.36) yields:

$$\pi^*_{P,t} = -\frac{\left(1-\sigma_G\right)\Lambda_y}{\kappa\lambda\Lambda_\pi}\left(\hat{y}^*_t - \hat{y}^*_{t-1}\right). \quad (F.38)$$

Combining Eqs.(F.37) and (F.38) yields:

$$\begin{aligned}
\pi^W_t &= -\frac{\left(1-\sigma_G\right)\Lambda_y}{\kappa\lambda\Lambda_\pi}\left(\hat{y}^W_t - \hat{y}^W_{t-1}\right), \\
\pi^R_t &= -\frac{\left(1-\sigma_G\right)\Lambda_y}{\kappa\lambda\Lambda_\pi}\left(\hat{y}^R_t - \hat{y}^R_{t-1}\right),
\end{aligned}$$

which correspond to Eqs.(22) and (25) in the text, respectively. Thus, the optimality conditions for self-oriented fiscal authorities are the same as the one under the optimal monetary and fiscal policy regime. This implies that the social loss is the same between the optimal monetary and fiscal policy regime under the cooperative setting and the self-oriented fiscal authorities with optimal monetary policy.

The definitions of the composite cost push terms imply the following:

$$\begin{aligned}
\varepsilon^2_t &= \Omega_9\xi^2_{H,t} + \Omega_{10}\xi^2_{\mathcal{N},t} + \Omega_{11}\xi^2_{G,t} \\
\left(\varepsilon^*_t\right)^2 &= \Omega_9\xi^2_{F,t} + \Omega_{10}\left(\xi^*_{\mathcal{N},t}\right)^2 + \Omega_{11}\left(\xi^*_{G,t}\right)^2,
\end{aligned}$$

with $\Omega_9 \equiv \left[\kappa\left(1+\varphi\right)\Omega_3\gamma\right]^2$, $\Omega_{10} \equiv \left[\kappa\left(1+\varphi\right)\Omega_3\left(1-\gamma\right)\right]^2$ and $\Omega_{11} \equiv \left(\kappa\sigma_G\Omega_4\right)^2$.

Substituting these equalities into Eqs.(F.15), (F.20) to (F.22) yields:

$$\begin{aligned}
\hat{y}^2_t &= \left(\Omega_8\Psi_1\right)^2\sum_{k=0}^{\infty}\Psi_1^{2k}\left(\Omega_9\xi^2_{H,t-k} + \Omega_{10}\xi^2_{\mathcal{N},t-k} + \Omega_{11}\xi^2_{G,t-k}\right) \\
\pi^2_{P,t} &= \Psi_1^2\left(1-\Psi_1\right)^2\sum_{k=0}^{\infty}\Psi_1^{2(k-1)}\left(\Omega_9\xi^2_{H,t-k} + \Omega_{10}\xi^2_{\mathcal{N},t-k} + \Omega_{11}\xi^2_{G,t-k}\right)
\end{aligned}$$

$$
\begin{aligned}
& - (1-2\Psi_1)\left(\Omega_9 \xi_{H,t}^2 + \Omega_{10}\xi_{\mathcal{N},t}^2 + \Omega_{11}\xi_{G,t}^2\right) \\
(\hat{y}_t^*)^2 &= (\Omega_8\Psi_1)^2 \sum_{k=0}^{\infty} \Psi_1^{2k}\left[\Omega_9\xi_{F,t-k}^2 + \Omega_{10}\left(\xi_{\mathcal{N},t-k}^*\right)^2 + \Omega_{11}\left(\xi_{G,t-k}^*\right)^2\right] \\
\left(\pi_{P,t}^*\right)^2 &= \Psi_1^2 (1-\Psi_1)^2 \sum_{k=0}^{\infty} \Psi_1^{2(k-1)}\left[\Omega_9\xi_{F,t-k}^2 + \Omega_{10}\left(\xi_{\mathcal{N},t-k}^*\right)^2 + \Omega_{11}\left(\xi_{G,t-k}^*\right)^2\right] \\
& - (1-2\Psi_1)\left[\Omega_9\xi_{F,t}^2 + \Omega_{10}\left(\xi_{\mathcal{N},t}^*\right)^2 + \Omega_{11}\left(\xi_{G,t}^*\right)^2\right] \qquad \text{(F.39)}
\end{aligned}
$$

Substituting Eq.(F.39) into Eq.(20) in the text, we have:

$$
\begin{aligned}
L_t^W &= \frac{\Psi_1}{2(1-\Psi_1^2)}\left[\frac{\Lambda_y\Omega_8\Psi_1}{2} + \Lambda_\pi(1-\Psi_1)\right]\{\Omega_9[\text{var}(\xi_{H,t}) + \text{var}(\xi_{F,t})] \\
& +\Omega_{10}\left[\text{var}(\xi_{\mathcal{N},t}) + \text{var}\left(\xi_{\mathcal{N},t}^*\right)\right] + \Omega_{11}\left[\text{var}(\xi_{G,t}) + \text{var}\left(\xi_{G,t}^*\right)\right]\},
\end{aligned}
$$

where we take the expectation in period zero on both sides. Substituting this equality into Eq.(19) in the text yields:

$$
\begin{aligned}
\mathcal{L}^W &= \frac{\Psi_1}{(1-\delta)2(1-\Psi_1^2)}\left[\frac{\Lambda_y\Omega_8\Psi_1}{2} + \Lambda_\pi(1-\Psi_1)\right]\{\Omega_9[\text{var}(\xi_{H,t}) + \text{var}(\xi_{F,t})] \\
& +\Omega_{10}\left[\text{var}(\xi_{\mathcal{N},t}) + \text{var}\left(\xi_{\mathcal{N},t}^*\right)\right] + \Omega_{11}\left[\text{var}(\xi_{G,t}) + \text{var}\left(\xi_{G,t}^*\right)\right]\}. \qquad \text{(F.40)}
\end{aligned}
$$

Substituting Eq.(F.39) into Eq.(26) in the text, we have:

$$
\begin{aligned}
L_t^{NC} &= \frac{\Psi_1}{2(1-\Psi_1^2)}\left[\frac{\Lambda_y\Omega_8\Psi_1}{2} + \Lambda_\pi(1-\Psi_1)\right][\Omega_9\text{var}(\xi_{H,t}) + \Omega_{10}\text{var}(\xi_{\mathcal{N},t}) \\
& +\Omega_{11}\text{var}(\xi_{G,t})],
\end{aligned}
$$

where we take the expectation in period zero on both sides. Substituting this equality into the definition of the respective loss in country H in the text yields:

$$
\begin{aligned}
\mathcal{L}_t^{NC} &= \frac{\Psi_1}{(1-\delta)2(1-\Psi_1^2)}\left[\frac{\Lambda_y\Omega_8\Psi_1}{2} + \Lambda_\pi(1-\Psi_1)\right][\Omega_9\text{var}(\xi_{H,t}) + \Omega_{10}\text{var}(\xi_{\mathcal{N},t}) \\
& +\Omega_{11}\text{var}(\xi_{G,t})].
\end{aligned}
$$

Substituting this equality and its counterpart in country F into the definition of the union-wide social loss brought about by self-oriented fiscal authorities in both countries, we have:

$$
\begin{aligned}
\mathcal{L}^{NCW} &= \frac{\Psi_1}{(1-\delta)2(1-\Psi_1^2)}\left[\frac{\Lambda_y\Omega_8\Psi_1}{2} + \Lambda_\pi(1-\Psi_1)\right]\{\Omega_9[\text{var}(\xi_{H,t}) + \text{var}(\xi_{F,t})] \\
& +\Omega_{10}\left[\text{var}(\xi_{\mathcal{N},t}) + \text{var}\left(\xi_{\mathcal{N},t}^*\right)\right] + \Omega_{11}\left[\text{var}(\xi_{G,t}) + \text{var}\left(\xi_{G,t}^*\right)\right]\},
\end{aligned}
$$

which implies that $\mathcal{L}^W = \mathcal{L}^{NCW}$. Furthermore, this equality and Eq.(F.40) correspond to the equality on page 24 of the text.

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Table 1: Macroeconomic Volatility in the Case where All Goods are Tradables ($\gamma = 1$)

Variable	Regime	Shocks			
		$a_{H,t}$	$a_{\mathcal{N},t}$	g_t^W	g_t^R
$\hat{y}_t^W$	OMP	2.5471e-004	0.0000	0.0013	0.0000
	OMFP	2.5471e-004	0.0000	0.0013	0.0000
π_t^W	OMP	1.3760e-005	0.0000	6.0145e-005	0.0000
	OMFP	1.3760e-005	0.0000	6.0145e-005	0.0000
$\hat{y}_t$	OMP	0.0070	0.0000	0.0013	0.0083
	OMFP	5.0941e-004	0.0013	0.0013	0.0078
$\hat{y}_t^*$	OMP	0.0065	0.0000	0.0013	0.0083
	OMFP	0.0000	0.0000	0.0013	0.0078
$\pi_{P,t}$	OMP	0.0080	0.0000	6.0145e-005	0.0301
	OMFP	2.7784e-005	0.0000	6.0145e-005	3.6616e-004
$\pi_{P,t}^*$	OMP	0.0080	0.0000	6.0145e-005	0.0301
	OMFP	5.3753e-006	0.0000	6.0145e-005	3.6616e-004
q_t	OMP	0.0000	0.0000	0.0000	0.0000
	OMFP	0.0000	0.0000	0.0000	0.0000
$\hat{r}_t$	OMP	0.0119	0.0000	0.0178	0.0000
	OMFP	6.8816e-004	0.0000	0.0016	0.0000
b_t	OMP	0.0000	0.0000	0.0000	0.0000
	OMFP	0.0135	0.0000	0.0162	0.0088
b_t^*	OMP	0.0000	0.0000	0.0000	0.0000
	OMFP	0.0088	0.0000	0.0162	0.0088

Notes:
OMP: Optimal monetary policy alone
OMFP: Optimal monetary and fiscal policy

Table 2: Macroeconomic Volatility in the Benchmark Case ($\gamma = 0.5$)

Variable	Regime	Shocks			
		$a_{H,t}$	$a_{\mathcal{N},t}$	g_t^W	g_t^R
$\hat{y}_t^W$	OMP	1.2655e-004	1.5016e-004	0.0013	0.0000
	OMFP	1.2655e-004	1.5016e-004	0.0013	0.0000
π_t^W	OMP	6.8366e-006	7.0097e-006	6.0145e-005	0.0000
	OMFP	6.8366e-006	7.0097e-006	6.0145e-005	0.0000
$\hat{y}_t$	OMP	0.0018	0.0022	0.0013	0.0047
	OMFP	2.5391e-004	2.9940e-004	0.0013	0.0078
$\hat{y}_t^*$	OMP	0.0016	0.0020	0.0013	0.0047
	OMFP	8.0096e-007	9.2692e-007	0.0013	0.0078
$\pi_{P,t}$	OMP	0.0081	0.0121	6.0145e-005	0.0408
	OMFP	1.4054e-005	1.4184e-005	6.0145e-005	3.6616e-004
$\pi_{P,t}^*$	OMP	0.0081	0.0121	6.0145e-005	0.0408
	OMFP	2.4640e-006	2.2170e-006	6.0145e-005	3.6616e-004
q_t	OMP	0.0035	0.0048	0.0000	0.0105
	OMFP	5.4188e-006	5.8689e-006	0.0000	0.0128
$\hat{r}_t$	OMP	0.0059	0.0074	0.0178	0.0000
	OMFP	3.4418e-004	3.8585e-004	0.0016	0.0000
b_t	OMP	0.0000	0.0000	0.0000	0.0000
	OMFP	0.0068	0.0088	0.0162	0.0066
b_t^*	OMP	0.0000	0.0000	0.0000	0.0000
	OMFP	0.0044	0.0052	0.0162	0.0066

Notes:
OMP: Optimal monetary policy alone
OMFP: Optimal monetary and fiscal policy

Figure 1: IRFs under Optimal Monetary Policy Alone in the Case where All Goods Are Tradable

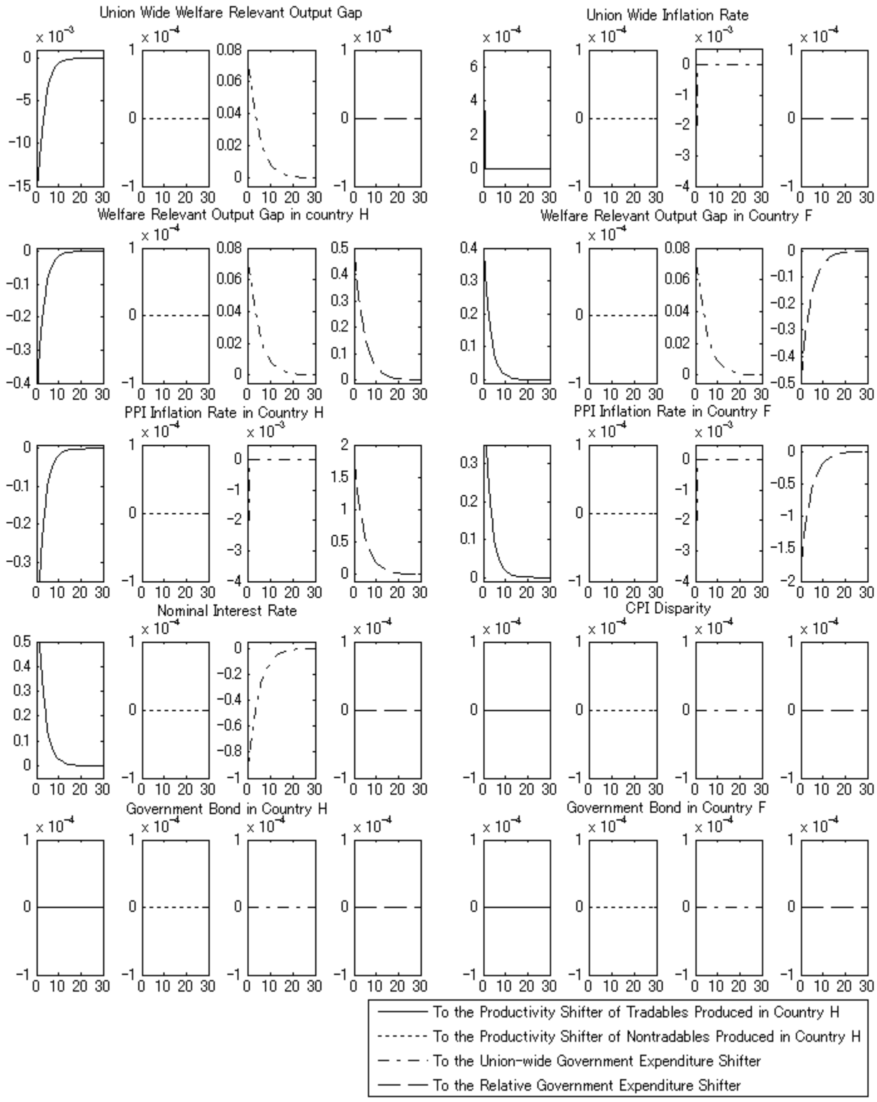

Figure 2: IRFs under Optimal Monetary Policy Alone in the Benchmark Case

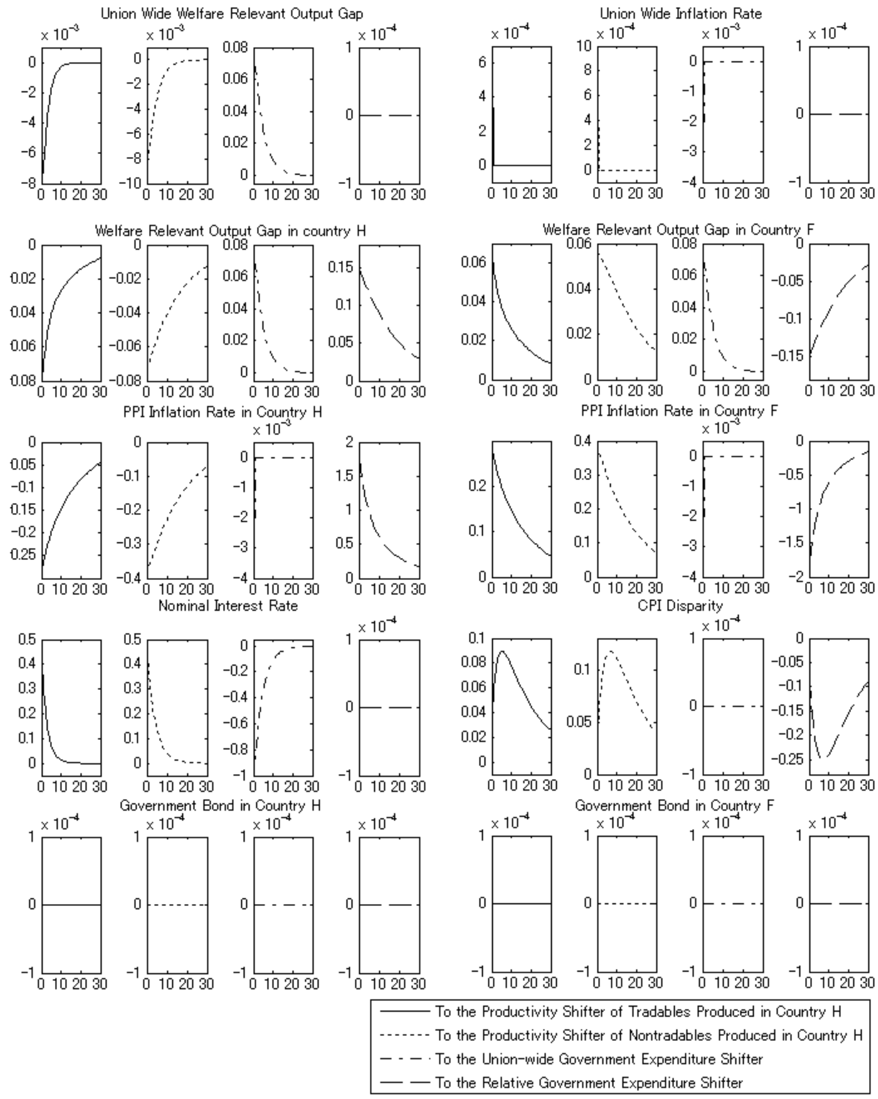

Figure 3: IRFs under Optimal Monetary Policy and Fiscal Policy in the Case where All Goods Are Tradable

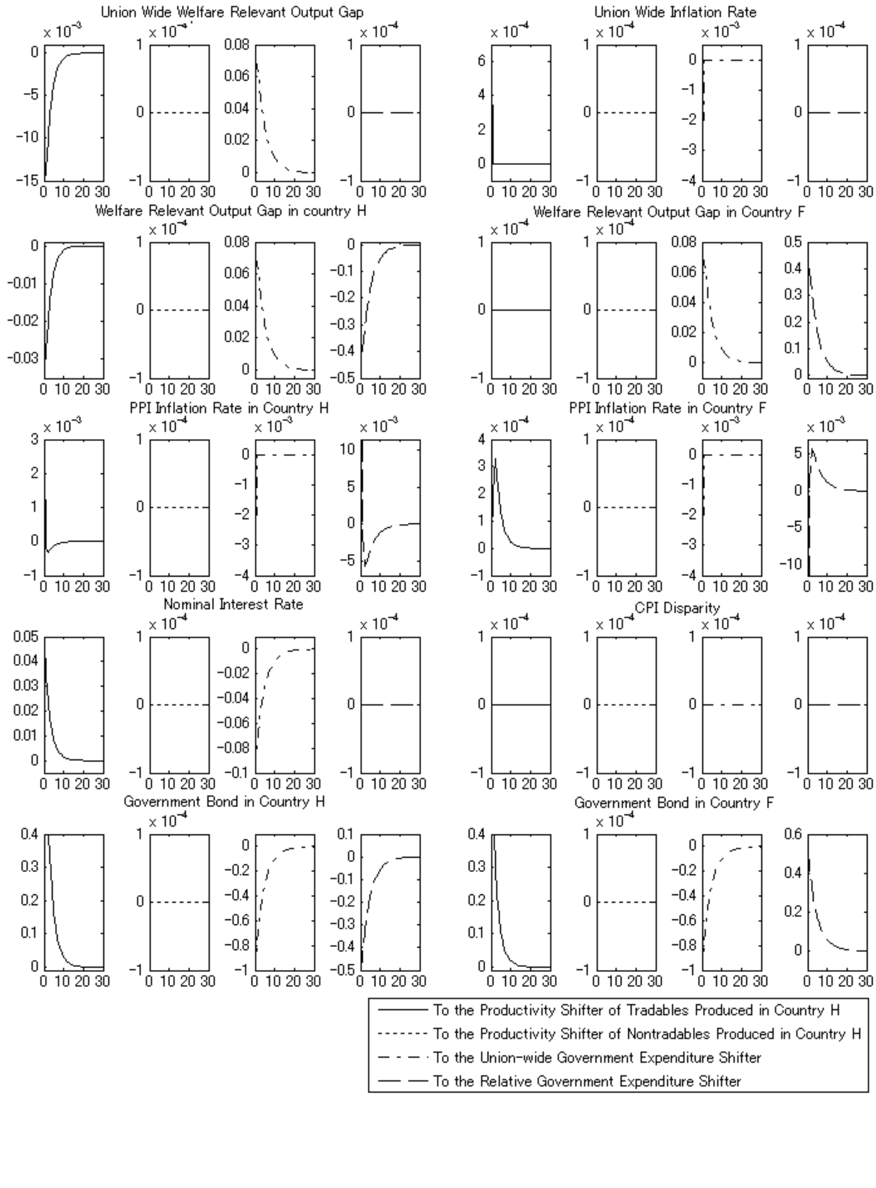

Figure 4: IRFs under Optimal Monetary Policy and Fiscal Policy in the Benchmark Case

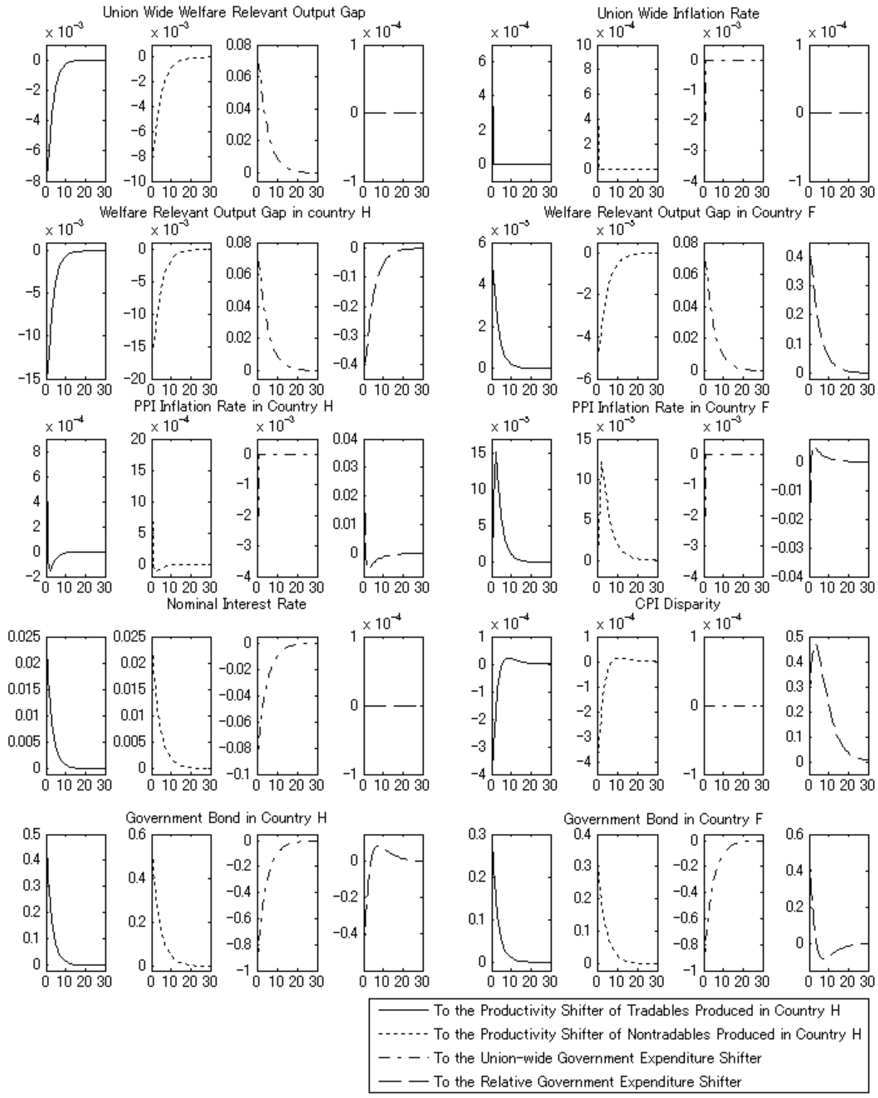

Figure 5: Effect on Welfare of Varying Shares of Tradable Goods